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ABSTRACT

Provided in this document is a brief summary of current research on equity in mathematics, readings on the topic, and lists of selected programs and resource materials. Readings presented include: "Teaching Mathematics in a Multicultural Setting: Some Considerations when Teachers and Students are of Differing Cultural Backgrounds" (Willis N. Johnson); "The State of the Art of Native American Mathematics Education" (Claudette Bradley); "Language Minority Students and Mathematics Achievement: The Language Factor" (Gilbert J. Cuevas); "Women and Mathematics: State of the Art Review" (Elizabeth Fennema); "Blacks in Mathematics: The State of the Art" (Martin L. Johnson); "Hispanic Students and Mathematics: Research Findings and Recommendations" (Leonard A. Valverde); and two ERIC Clearinghouse for Science, Mathematics, and Environmental Education fact sheets ("Females and Mathematics" and "Helping Low-Achieving Students in Mathematics"). Programs described include Equals, Math Anxiety Programs, Math Counseling Institute, "I Can't Do Math", Overcoming Math Anxieties, Improving Teachers' and Counselors' Mathematics Abilities and Attitudes, Meeting the Math Challenge for the 80's, Mathematics Their Way (Workshop), Mathematics...A Way of Thinking (Workshop), Multiplying Options and Subtracting Bias, and Project Seed. Assresses, contact persons, state, institution, and materials available are included with each description. Resources listed and Project Seed. Addresses, contact persons, state, institution, and materials available are included with each description. Resources listed include books, annotated journal articles, RIE abstracts (with ED numbers), associations, career information, and other materials. (JN)

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A RESOURCE BOOK FOR EQUITY IN MATHEMATICS

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A RESOURCE BOOK FOR EQUITY IN MATHEMATICS

Table of Contents

	Page No.
Summary of Current Research	1
Selected Readings	
Johnson, Willis N., "Teaching Mathematics in a Multicultural Setting: Some Considerations When Teachers and Students are of Differing Cultural Backgrounds"	5
Bradley, Claudette, "The State of the Art of Native American Mathematics Education"	15
Cuevas, Gilbert J., "Language Minority Students and Mathematics Achievement: The Language Factor"	25
Fennema, Elizabeth, "Women and Mathematics: State of the Art Review"	35
Johnson, Martin L., "Blacks in Mathematics: The State of the Art"	53
Valverde, Leonard A., "Hispanic Students and Mathematics: Research Findings and Recommendations"	69
ERIC, "Females and Mathematics"	81
ERIC, "Helping Low-Achieving Students in Mathematics"	83
Selected Programs	85
Resources: Books, Articles, ERIC Documents, Other Resources	101

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SUMMARY OF CURRENT RESEARCH

Because it is the mission of the public schools to provide equal access to education for all children, concern has been increasing in the last few years over the under-representation of women and minority students in mathematics classes and subsequently in careers which require knowledge of mathematics.

In reviewing the academic preparation of students in their first year at the University of California at Berkeley in 1972, researcher Lucy Sells found that 92% of women freshmen, as opposed to 43% of the men, had not taken four years of high school mathematics. Since three-fourths of the majors at the university required such preparation in mathematics, the options for 92% of the incoming women freshmen were severely limited. This picture is repeated across the nation.

Sells applied the term "critical filter" to the study of mathematics because of its significance in determining whether women will be segregated out of higher paying, higher status careers. Avoidance of mathematics and poor mathematics achievement are thus seen as contributing to inequity and discrimination for women and minority students, both during the years of their education and in their later lives.

Efforts to counter the effects of differential treatment, and to encourage female and minority students to participate and achieve in mathematics, include the identification of 1) successful strategies for teaching, and 2) exemplary mathematics programs which can be recommended for use in schools.

The United States has taken the lead in research about the relationship of gender to mathematics achievement, despite the recognition of similar disparities in achievement in other nations. Generally, there has been much less available research on factors governing mathematics achievement among minority children, or replication of majority research with minority children, in any country.

Continuing the pattern of findings in the 1970's, current research on sex differences in mathematics achievement in the U.S. indicates that these differences persist, even, in some cases, when students have taken the same amount of coursework. Sex differences in achievement are still found from junior high school age through college.

These differences, however, are not shown to result from inherent factors or from gender-based differences in visual/spatial ability. Some studies indicate that visual and spatial ability each "contribute to" predicting math achievement, but definitions of visual and spatial abilities are not consistent and the relationship between them and mathematical learning is not clear.

Factors in low mathematics achievement by minority students have been identified as: lingusitic effects, school influences and learning style/personality/self-concept. Concern has been expressed over non-participation in mathematics study by Hispanic, Black and Native American children, and attention has been drawn to the bilingual needs of many minority children in this as in other school subjects.

Increasing awareness of the role of students' anxiety/confidence in mathematics achievement has resulted from studies showing the influence of teachers, peers, socialization and mathematics preparation on the confidence level of mathematics students, with special effects related to differential treatment of male and female students. Boys continue to be more confident of their mathematics ability than girls.

Mathematics is still stereotyped as a "male domain." Differential treatment of girls and boys is observed in parents, who have lower mathematics expectations of daughters than of sons, and in teachers, who expect differences between the sexes and treat students differently on that basis. Students themselves then respond differently, to meet those expectations. Even "extremely talented" girls tested in one study of seventh grade mathematics students had not been identified early by either parents or teachers, whereas their male counterparts had been.

SELECTED READINGS

Teaching Mathematics in a Multicultural Setting: Some Considerations When Teachers and Students are of Differing Cultural Backgrounds.

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This presentation will not deal with: a) specific discipline or behavior problems that may arise in a multicultural/multi-ethnic/bilingual setting--however, this discussion may have a direct influence on discipline or behavior; b) nor will this presentation identify the differences that may exist between subcultures and ethnic groups when considering mathematics learning.

This presentation will attempt to illustrate that mathematics in the elementary grades is not and should not be "culture free." As a prescriptive course, the discussion will give suggestions on how to improve the mathematics experience of elementary school children by using the culture of both the teacher and the learner in mathematics instruction.

If one were to examine textbooks, psychomotor materials, perceptual materials, audio-visual materials, and other commercially produced materials for the learning of mathematics at the elementary school level, one will see pictures, word problems, etc. that depict a certain occupation, sex, appearance, name, places, events, voice quality, etc. "common" to the majority of Americans. After so much culturally related evidence, can one say the mathematics learning is "culture free?" Hardly. But, the question is whose "culture" is being exemplified?

Banks (1975) discusses culture along a pluralistic--assimilationistic continuum. Let's look at some assumptions about the aspects of culture and learning.

<u>MATCH THE STATEMENTS BELOW WITH EITHER THE PLURALISTIC, PLURALISTIC--ASSIMILATIONISTIC, OR THE ASSIMILATIONISTIC POINTS OF VIEW:</u>

Statement One: Cultural subgroups have unique cognitive styles.

Statement Two: Cultural subgroups have some unique cognitive styles, but share many learning characteristics with other groups.

Statement Three: Human learning styles and characteristics are universal.

Regarding culture and learning, statement one is the view of the cultural plurlist. Statement two is the point of view of the cultural pluralist--assimilationist. Statement three, then, is the view of the pure cultural assimilationist. (The reader is to assume the pluralist and the assimilationist to be at opposite ends of the acculturating continuum.)

To further establish the link between learning styles and culture, Ramirez and Castaneda report three styles:

A. Incentive-motivational styles that are short and long term goal oriented.

B. Human relational styles that are concerned with internal and external locus of control.

and C. Patterns of Intellectual abilities and learning styles which deal with mechanisms for collecting, organizing, and using information about the environment (i.e. inductive (deductive)" (Ramirez and Castaneda, 1974)

In the report by Ramirez and Casteneda they further report two identifiable categories into which individuals (and in this case, cultural subgroups) would distinctly fall--field dependent learners and field independent learners.

A. Field-dependent children do best on verbal tasks of intelligence test; learn materials more easily which have humor; are sensitive to the opinions of others; perform better when authority figures express confidence in their ability; and, conversely, perform less well when authority figures doubt their ability.

B. Field-independent children do best on analytic tasks; learn material that is inanimate and impersonal more eagerly; and their performance is not greatly affected by the opinions of others." (Ramirez and Castaneda, 1974)

One of the dangers of classifying learners into such learning styles is the tendency to make learning somewhat "half-witted"--especially when one considers the overwhelming evidence that relates the field dependent and field independent behaviors with that of the nemispheric regions of the brain.

Another danger implied from these research findings, is the notion teaching styles usually fall quite clearly into field-independent and field-dependent behaviors. Clearly, there will be the possibility of half-witted teaching or erratic communication of ideas to be learned.

A listing of learner behaviors paired with teacher behavior is provided for your study.

Field Dependent	Field Independent
Learner Behaviors	
Relationship to peers.	Relationship to peers.
1. Likes to work with others to achieve a common goal.	1. Prefers to work independently.
2. Likes to assist others.	2. Likes to compete and gain individual recognition.
3. Is sensitive to feelings and opinions of others.	3. Task oriented; is inattentive to social environment when working.
Personal Relationship to teacher.	Personal Relationship to teacher.
1. Openly expresses positive feelings for teacher.	1. Rarely seeks physical contact with teacher.
2. Asks questions about teacher's tastes and personal experiences; seeks to become like teacher.	2. Formal interactions with teacher are restricted to tasks at hand.
Instructional Relationship to teacher.	Instructional Relationship to teacher.
1. Seeks guidance and demonstration from teacher.	1. Likes to try new tasks without teacher's help.
2. Is highly motivated when working individually with teacher.	2. Impatient to begin tasks; likes to finish first.
	3. Seeks nonsocial rewards.
Teacher Behaviors	
Personal behaviors.	Personal behaviors.
1. Displays physical and verbal expressions of approval and warmth.	1. Maintains formal relationship with students.
2. Uses personalized rewards which strengthen the relationship with students.	2. Centers attention on instructional objectives; gives social atmosphere secondary importance.
Instructional behaviors.	Instructional behaviors.
1. Expresses confidence in child's ability to succeed.	1. Encourages independent student achievement.
2. Gives guidance to students; makes purpose and main principles of lesson obvious to students.	2. Encourages competition between students.
3. Encourages learning through modeling; asks children to imitate.	3. Adopts a consultant role.
4. Encourages cooperation and development of group feelings.	4. Encourages trial and error learning.
5. Holds internal class discussions relating concepts to student's experiences.	5. Encourages task orientation.
Curriculum related behaviors.	Curriculum related behaviors.
1. Emphasizes global aspects of concepts; clearly explains performance objectives.	1. Focuses on details of curriculum materials.
2. Personalized curriculum.	2. Focuses on facts and principles; encourages novel approaches to problem-solving.
3. Humanizes curriculum.	3. Relies on graphs, charts and formulas.
4. Uses teaching materials to elicit expression of feelings from students.	4. Emphasizes inductive learning and discovery approaches.

With the foregoing characteristics in mind about learning styles, let's examine some teaching assumptions from a cultural point of view. Choose the one that best represents your point of view.

Statement One: Students need skilled teachers who are very knowledgeable about and sensitive to their ethnic cultures and cognitive styles.

Statement Two: Students need skilled teachers of their same race and ethnicity for role models, to learn more effectively, and to develop more positive self-concepts and identities.

Statement Three: A skilled teacher who is familiar with learning theories and is able to implement those theories effectively is a good teacher for any group of students, regardless of their ethnicity, race, or social class. The goal should be to train good teachers of children.

Statement one, two, and three respectively are the points of view of the cultural pluralistic-assimilationist, the cultural pluralist, and the assimilationist.

So far we have looked at learning styles and desired teacher behaviors. Now let's turn to the cultural assumptions that relate to selecting, designing, and the use of mathematics, curricula materials. First from the statements given below, choose the one that best represents your point of view.

Statement One: Use materials and teaching styles which are related to the common-culture. The curriculum should help the child to develop a commitment to the common civic culture and its idealized ideologies.

Statement Two: Use materials and teaching styles which are culture specific. The goal of the curriculum should be to help the child to function more successfully within his or her own ethnic culture and help to liberate his or her ethnic group from oppression.

Statement Three: The curriculum should respect the ethnicity of the child and make use of it in positive ways. The goal of the curriculum should be to help the child learn how to function effectively within the common culture, his or her ethnic culture, and other ethnic cultures.

Statements one, two, and three respectively are the points of view of the cultural assimilationist, the cultural pluralist, and the cultural pluralist-assimilationist.

In order to enrich the mathematics curriculum with the culture of the learner, it was the intent of this paper to argue for the cultural pluralist-assimilationist point of view. Consequently, it is imperative to draw examples and procedures from both the "common" culture of the United States as well as from the "local" culture from which the learner comes. The development and reinforcing of understanding of mathematics concepts should utilize the notion of "social exchange" and not social power to lead to productivity within the classroom. (Larking, 1975)

The use of instructional materials in the teaching of mathematics should follow this paradigm:

1. Concrete and semiconcrete materials: Use materials that are from the common culture plus those specific to the learners in the class.

2. Abstract materials: Encourage learners to interpret abstraction in the form of story situations they contrive.

3. Applications: Use occupations, foods, places, and events that are in the learners environment.

4. Drill and Practice: Since much time is spent here, use a variety of social and cultural exchanges wherever possible.

Some sample activities on which you might build are:

Objective: Match numerals 1 through 10 with sets of objects.

Materials: Posterboard, stickers or small pictures, felt pen, clothespin.

Procedure: Ask the learner to count the pictures in one set on the posterboard and attach the corresponding numeral which will be on the clothespin to that set.

Given the numeral, ask the learner to find the corresponding set.

Culture Specific:	Use pictures for sets that relate to a particular cultures, or to a holiday.
Objective:	Addition or subtraction with single digit numbers.
Materials:	Baking pan, cardboard, marker.
Procedure:	Draw and cut out 10 figures in pan. Learner can add or subtract the number in and/or out of the pan.
Culture Specific:	The figures could represent tortillas, hotcakes, cornbread, etc.
Objective Materials:	Reinforcing basic operations (facts), Glue, sturdy paper or board (2 pieces), a picture.
Procedure:	Glue the picture on one piece of paper or board. Cut the picture into puzzle like parts and write problems on each puzzle part. Write the answers on the second piece of paper or board as they would appear or as they should appear when the puzzle is worked out.
Culture Specific:	Use a picture appropriate to the child's culture.

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The State of the Art of Native American Mathematics Education

Claudette Bradley
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The State of the Art of Native American Mathematics Education

Today Native Americans are developing their communities and reservations to maintain their rights, lands, and people. In an attempt to gain equity in a world of computer and high technology industries, Indians need equal education, particularly in mathematics. Their communities and reservations are increasingly seeking to hire Indians trained in vocations and professions, which require competence in mathematics (Green, 1978, p. 13).

"Of all minority groups in the United States, Native Americans are the most poorly represented in the natural sciences, the health sciences, and mathematics; yet Native peoples are the most needful of any minority group of status improvement in the areas of health, education and social welfare. Indian health needs are generally well-known, as are the grim facts of Indian underachievement in education. Even though the necessity for more Indian professionals in all fields and for economic development throughout the Native world tie together the major areas of concern, other needs beyond basic health care delivery and education exist in plenty. Most land-based Native Americans are in a serious struggle for the retention and development of the land they own, and in that struggle lies a potential for economic development which can change health care and education patterns for the good. Thus, Native engineers, geologists, agronomists, aquaculture specialists, chemists, geneticists, animal husbandry specialists, and botanists--are all needed for resource development, management, and planning on Indian lands. Additionally, Indian teachers and programs in these fields are needed to raise the general educational level of Native people, as well as to provide meaningful career options for all Indian--rural, urban, land-based and not." (Green, 1978, p.1)

Achievement scores of Indian students fall below grade level as they progress through elementary and secondary schools. Indian students avoid high level mathematics courses in high school. Upon entering a college or university they are often unprepared for the calculus sequence, and select majors with little or no mathematics requirement. The result is very few Indians enter mathematics related careers.

About 70% of Native American students attend public schools in the United States. The remainder attend federal BIA supported schools, private, and parochial schools. The 1970 US Census found 2,829 school districts with at least one Indian student. These districts averaged 3% Indian. Of those enrolled in public schools 70% live in five states: California, Arizona, New Mexico, Oklahoma, and Alaska. (See table 1) Montana, South Dakota, and the State of Washington each have more than 20 districts with over 10% Indian enrollment (Scherbeck, 1976).

New Mexico has 23,964 Indian students enrolled in public schools allocated among 15 school districts. On tenth grade Proficiency Examination given to all students in New Mexico, Indian students had the lowest percentage scoring at or above 65% correct level when compared to Blacks, Hispanics, and Anglos (Southwest Resource Center for Science and Engineering, 1981, p. 7). (See table 2)

The State of Washington has 18,114 Indian students enrolled in public schools. 4,407 Indian students are enrolled in districts of at least 10% Indian enrollment. These students are allocated in 25 school districts (Scherbeck, 1976). The Johnson O'Maley (JOM) programs compiled mathematics achievement test scores for Indian students in Washington. (See table 3)

Choctaw Indian students in Mississippi scored below grade level in mathematics on the California Achievement tests. The gap between the mean teast scores and the grade continues to widen as the students progress from 2nd to 12th grade. By 12th grade Choctaw Indian students' mean is at 8th grade level in mathematics (Brod, 1979, p. 13). (See table 4)

Sells (1973) discloses high school mathematics is a critical filter for anyone entering the job market or seeking higher education. Students who have not taken Algebra and Geometry in high school are ineligible for admissions to some colleges upon graduation (Office of Provost, 1976). Students without a background in trigorometry are not prepared for the freshman calculus sequence in college and must limit their choice of undergraduate major to education, social sciences, arts and humanities, if they plan to continue to avoid mathematics courses. Students who choose to enter vocations such as surveyors, auto mechanics, machinists, carpenters, roofers, electronics workers and technicians in scientific and industrial laboratories, are not required to earn college degrees, but are required to study high school mathematics beyond algebra (Krienberg, 1976, p.2).

The evidence shows Native Americans are not obtaining sufficient competence in mathematics to study the higher level mathematics courses in high school, to take the calculus sequence, and to enter mathematics related careers. All the recruitment efforts of Indian communities and reservations together with high technology industries to employ Indian people in careers requiring mathematics or statistics will be unsatisfactory. As long as Indian students avoid mathematics and limit their choices in the job market, there is little hope to raise the overall relative family incomes of Indian people to any significant degree.

If mathematics educators of Indian students are to resolve this dilemma, we must investigate the contributing factors. For any school setting the student's education is effected by teachers, curriculum, administration and community. Indian students have the added dimension of living in two worlds-home and school which are products of two cultures - Indian and White. The questions are: How do these cultures contribute to Indian students avoidance of mathematics and lack of competence with mathematics? And what other factors in the school setting contribute to the avoidance and lack of competences?

Dr. Rodney Brod (1976) compared Indian students with their classmates and found the differences to be that Indian students had more resident changes, were more rural, lived further from school, rode the school bus, had older siblings and more sisters, had large families, and no phone listed in school files (Brod, 1976). To consider these factors, no phone, changing residence, and living far from school, teachers and school administrator would not have an easy time contacting parents when they needed. Likewise, having large families, living far from school and no phone, parents would not have an easy time visiting or contacting the school.

Indian students struggle to survive in two cultures. Generally teachers and school administrators are not Indian and have the cultural values of mainstream society. Indian students attend school 6 hours a day and bring the attitudes, values, and beliefs from their home environment and Indian communities into the classroom.

Charles G. Moore (1981) and Lehi Smith (1981) have written about mathematics learning of Navajo students in relation to the Navajo Language. They concur that the language has styles of thought and communication, which influence students in their approach to solving problems and learning mathematical concepts. Their papers indicate that research is needed to fully understand the effect of Indian Language on mathematics learning of Indian students.

Jack Easley (1980) in On Understanding the Mechanism of Underrepresentation of Minorities in Mathematics, says children of other cultures have different personal insights about behavior patterns (p. 6). Some decide to tell their feelings others attempt to keep their feelings. Elementary teachers of Indians remark about Indian children's silence in the classroom. This factor makes 'discovery' based teaching methods inappropriate in traditional Indian communities according to Green (1978, p. 5). However, Indian students' behavior of silence should only discount discovery based learning when verbal responses are expected. Discovery is an intimate part of everyone's learning. The expression of that learning does not have to be a verbal response, but can be non-verbal. Especially, since traditional Indian communities/tribes had highly sophisticated forms of non-verbal communication.

Reporting on the conference on Mathematics in American Indian Education, Rayna Green (1978) writes, "conferees agreed many mathematics teachers and counselors perpetuated the mystique of math as hard and inaccessible to all, but the brightest students" (p. 4). This may occur from their own feelings about mathematics, for they suffer from painful experiences with math (Mitzman, 1976, p.11).

Conferees on Mathematics in American Indian Education agreed Indian students are both unprepared and dissatisfied with mathematics (Green, 1978, p. 4). Indian teenagers are more verbal than those in elementary school. Teenagers rebel against the "stringent discipline practiced by federal and parochial schools" and choose not to study the most "disciplined, visible Western forms of education", which they believe to be math and science courses (Green, 1978). In high school Indian students select social science and humanities, for they appear more relevant to their lives.

Teachers and counselors believe Indian students are incompetent in mathematics. They encourage only the most promising students to take higher mathematics courses in high school. Dr. Brod, University of Montana, found teachers will give white students the benefit of the doubt when grading, but seldon extend this practice to Indian students (Brod, 1976).

Parents influence teenagers, expecially mothers. In Choctaw student assessment of vocational needs the question was asked: "In choosing your job or career, whose ideas are most important?" The major responses were 'My ideas' (81.9%) and 'mother's ideas' (46.9%). The major career choices of Choctaw student's were Professional (40.6%), and Craft/foreman (15.6%)

(Brod and Brod, 1979). In Colville student assessment of vocational needs the same question was asked. The major responses were "my ideas" (90.3%), and "mother's ideas" (52.2%). The major career choices of Colville student's were professional (32.1%), operative (11.2%), clerical (10.4%) and service worker (10.4%) (Brod and Brod, 1981).

Two types of students appear to be good candidates for mathematics based career. The first has an excellent memory for algorithm and definitions, plus has successfully practiced their application on classroom assigned problems. The second is the individual who reconstructs mathematical ideas, creates problems to test ideas, completes all classwork and seeks to learn more about mathematics by himself (Easley, 1980). Traditional Indians had a highly developed memory needed to pass on the culture (Moore, 1981). This ability to memorize has not been employed in the mathematics classroom. But as Easley writes, students who memorize definitions and algorithms do not survive in higher mathematics courses as well as the "self-taught problem solvers who easily create ways to solve new mathematics problems" (1980, p. 3). In any case neither of these two types of math students is being encouraged among Indian students by their teachers.

The question to address at this point is how teachers contribute to the lack of mathematical competence among Indian students?' In elementary schools teachers teach many subjects. They suffer from math anxieties developed in their own school days (Sells, 1973, p.2). Generally elementary school teachers are not trained nor enthused about mathematics. High school teachers are trained in the subject but lack special training and commitment for Indian students (Green, 1978). Most teachers believe in equal education for all, but they do not have the resources to meet the responsibility. Teachers criticize their studies at college and universities. None of their courses provided the understanding of problems they dealt with in the classroom (Easley, 1980).

Lucy Sells (1973) wrote "the traditional classroom forces people into proving other people wrong." Jack Easley (1980) wrote "Mathematics classes are 'extremely dull' in all grades K-12 in all kinds of schools for all kinds of students." When students simply memorize facts, they do not learn how to apply them in the real world. When they rely on formulas, they do not learn to think (Heltzel, 1976). The two forms of problem solving are: application of algorithms to written problems and the creative exploration of a problem situation. The first is considered problem solving by elementary school teachers. The second is needed in advanced mathematics courses and in real life situations (Easley, 1980). Elementary school teachers are afraid of students who demonstrate creativity in mathematics. They overlook the curiosity of students and prefer to focus on the economic pay-off in later life. Green (1978) concurs that Indian students are encouraged to study math only to fulfill a college requirement or vocational program.

Mathematics materials for Indian students needs more attention (Green, 1978). Many textbooks used by Indian students do not address their cultural and intellectual interests. The quality tends to be less than satisfacory (Gore, 1980).

Herbert Their, of Science Curriculum Improvement Study at UC, Berkeley, warns educators against the "remediation syndrome" and "identity accomplishment confusion." The "remediation syndrome" is the belief the child is the problem and needs more time at the same thing. By spending more hours the child suffers longer and develops a poor self-image and dislike of the subject.

The "identity accomplishment confusions" is teaching the child his own culture within the curriculum. Such efforts in the past resulted in changing illustrations or stories. Children spent too much time on traditions, songs and games, which would be okay if the school didn't side step teaching the skills needed to survive in todays world. Attempts to teach a culture by teachers without direct experience in that culture either programmed the child or caused him to respond passively. Dr. Their suggests the school should focus on the child's learning process and develop curriculum with the advise and cooperation of community members. The community should encourage children to obtain school related skills and relate their importance to their culture and community. Dr. Their feels an experience-based curriculum which integrates the Indian students' school, home, and other life experiences is the most promising approach in the mathematics classroom. The experienced based curriculum should be planned with the cooperation of school and Indian community.

Answers to why Indian students are not gaining mathematical competence are many. Solutions vary depending on the culture and environment of specific Indian students. However, mathematics educators should find the following programs and ideas significant for Indian mathematics education.

George Cardell (1978) wrote "Extending Counselor Influence into the Classroom ". He describes the peer learning experience used by mathematics teachers of Mascalero-Appache children. The teacher presented the mathematics lesson in the traditional fashion. Learning stations were set up around the classroom with a peer learning leader at each station. Students were free to choose their leader and to rotate among the stations. The leader reviewed the lesson, allowed students to ask questions, and checked their work. To wind up the lesson the learning leader gave graded check-up reviews which were spot checked by the teacher. When compared with a control group the students in the peer learner experience gained more in mathematical concepts and skills, developed positive attitudes toward the experience and demonstrated growth toward positive attitudes toward mathematics (Cardell, 1978, p.12).

John Early (1973) wrote "Education via Radio among Guatemalan Highland Maya". Thirty-four teachers converted their homes into classrooms three evenings a week. Core lessons were broadcasted each evening in the Tzutujil language. The teachers turned off the radio for review questions and for checking students work. Once a week the teacher returned to the broadcast office to discuss their classes and the lesson for the coming week. This program reached 906 students over four years. The students were friends and relatives of the teachers, but they had relatively no formal schooling. "In comparison with other educational and development projects in the Highland Maya area,...the results have been outstanding" (Early, 1973, p. 228).

Native American loom beadwork has properties which illustrate mathematical concepts in Geometry, Coordinate Geometry, Number, Theory, and Measurement. Claudette Bradley (1976) has used loom woven beadwork to develop mathematical culture based curriculum for Indian students. After students master the task of making loom beaded items the students were to develop their beadwork designs on computers using Log. Seymour Papert (1980) has developed Turtle Geometry using Logo computers. His theory is that students learn mathematics by using mathematics to write computer programs that draw shapes, designs, or animated pictures on the Logo TV-screen (Papert, 1980). Native American loom beadwork designs are ideal subjects for Logo computers in technique color.

Project SEED designed by William Johntz (1980) is for minority school age children. Trained scientists and mathematicians teach abstract conceptually oriented mathematics with the discovery method. The staff loves mathematics and communicates well with students. They have observed creative answers and questions in their students to responses that are generally considered "wrong" answers by traditional mathematics teachers. Project SEED has been successful in Berkeley, California with 4th and 5th grades for over 17 years.

Oregon Mathematics Learning Center Salem, Oregon has designed creative ways for teaching mathematics. One way is the lab approach (Mitzman, 1976, p. 11). The Mathematics lab provides opportunities for students to explore mathematics through games and hands on materials. The center offers workshops for teachers, provide a place for materials for learning mathematics. About a third of the teachers have been Indian. The written responses of teachers who attended the workshops were very positive. Many teachers became math enthusiasts (Mitzman, 1976; Heltzel, 1976).

In summary, to address the question of what factors may improve the mathematical competence of Indian students the analysis of this paper places the findings in three categories: the cognitive domain, the affective domain, and the social domain. In the cognitive domain Indian students must be encouraged to reconstruct mathematical ideas, create problems, test ideas, complete classwork, and seek to learn more mathematics on his own. As in the SEED project, Indian students need to develop spatial relationships, which can be accomplished through creative uses of hands-on materials, as well as, calculators and computers. In the affective domain Indian students need support and counseling from Indian community members and mathematics related professionals. In the social domain Indian students may need briefing on mathematical language, test-taking strategies, and/or appropriate problem solving techniques prior to studying higher math courses. Culture based mathematics should not push aside the main focus of learning mathematics, but would be most effective if Indian community members took primary responsibility of developing culture based mathematics both in and out of school and worked cooperatively with mathematics teachers of the school.

Mathematics educators should be aware of research in culturally appropriate communication styles, especially in reservation areas. Of particular interest should be the issue of silence in elementary schools of Indian student and the traditional sophisticated non-verbal forms of communication. There are other forms of expressing learning than talk. The use of hands-on materials and computers would be appropriate for quiet students. Discovery methods would work and teachers would learn to recognize learning.

At the high school level particularly, Indian educators should research the job market within Indian communities as well as the greater non-Indian community. They should familiarize themselves with the role of mathematics in these vocations and professions, relating the information to students, and creating opportunity for students to meet Indians in those fields.

To make any lasting effective change in mathematics education parents, tribal leaders, counselors, teachers, and administrators must be educated about the necessary changes. Mathematics has been taught the same way in elementary and secondary schools for over a hundred years. Educators learned from the intervention of "New Math" in the 1960's that when teachers and parents become frustrated over the changes in mathematics curriculum, the

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LANGUAGE MINORITY STUDENTS AND MATHEMATICS ACHIEVEMENT: THE LANGUAGE FACTOR

By

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The Problem

One of the basic problems we face as mathematics educators, deals with the ways in which we can be more effective in communicating mathematical ideas. This communication becomes critical when teaching students for whom English is a language not understood. In view of the learning difficulties which face language minority students, the lack of proficiency in English is found to be at the core of the problems. Language proficiency is also the focus of the various legislative and legal mandates which direct school systems to provide equal educational opportunities to linguistic minority groups. Both the Bilingual Education Act and the Lau vs. Nichols "Remedies" address the fact that ". . . students who do not understand English are effectively foreclosed from any meaningful education" (414 U.S. at 566, 1974). It is widely believed that the linguistic minority students' failure to achieve in school is caused by the "language problem." Research studies have also indicated that there appears to be a relationship between these children's degree of proficiency in English and academic functioning (Lewis, 1959; Carter, 1970; Anderson and Johnson, 1971; Campbell, 1973; De Avila, Cervantes and Duncan, 1970; Cummins, 1981). It may be assumed from these results that the more proficient in English these students are, the greater the opportunities available for them to achieve in school. Applying this assumption to the learning of mathematics two basic questions must be addressed before intervention programs are designed to assist these students. First, to what extent does the student's first and/or second language effect learning and the assessment of achievement in mathematics? Second, what English language skills are necessary for successful achievement in the subject?

The above questions provide the foundation for the study of the problem--underachievement in mathematics as a result of lack of proficiency in English. In addition, the issues raised by the questions are relevant to all language minority students; the inability to use the English language in a school setting is a problem all language groups face.

Analysis of the Problem

The language factor in mathematics education may be analyzed in terms of its effect on the learning of mathematics and its role in the assessment of achievement. Also, an analysis of the language skills which are needed for successful mastery of mathematical concepts and skills may contribute to a further clarification of the problem and possible solutions.

The interest in the relationship between language and learning is not a new one. Some have suggested that language determines and defines thought. Others, such as Piaget (1956), have tended to accept only a limited effect of language on thought. However, his coworkers have expanded on this premise by suggesting that language development may be influenced by general cognitive development since some major changes in language mastery occur at times when major cognitive changes occur. From a further review of the literature, it may be assumed that language and learning are strongly related.

The relationship between language mastery and mathematics learning has long been accepted, although not always recognized as a factor in the learning of mathematical concepts and skills. There is sufficient evidence of this in the literature. Thorndike (1912) stated that ". . . our measurement in arithmetic is a measure of two things: sheer mathematical knowledge on the one hand; and acquaintance with language on the other." Aiken (1971) in a review of verbal factors and mathematics learning, reports that researches have long recognized the role language plays in performance in mathematics. Studies indicate that there is a positive correlation between reading ability and scores on tests of problem solving in arithmetic. More recent studies performance and second language ability (Cossio, 1977). In addition, there appears to be a relationship between instruction through the student's native language in curriculum content areas and high achievement in those areas (Granado, 1976; Tsang, 1976; Coffland and Cuevas, 1979; Cuevas and Liabre, 1981). In summary, the relationship between language factors and mathematics achievement is not clearly understood but may be appropriate to assume that in order for a student to master the mathematics concepts, the language of the concepts must be mastered.

In addition to the relationship between language and the learning of mathematics, the role language plays in the assessment of mathematics achievement must be discussed. The utilization of a language not understood by students is frequently given as one of the reasons for the inappropriateness of existing tests and testing situations when used with these students (Moreno, 1970; De Avila and Havassy, 1974; Ramirez and Gonzales, 1972.) According to Chandler and Plakos vocabulary contained in the instrument is unfamiliar to them. Gearder (1975) provides an argument supporting the notion that the use of English as the language of the tests is one reason for the obtained decreases in the achievement scores of Hispanic students. Although this claim is debatable, several studies have lent support to this hypothesis. Holland (1960), Mycue (1968), and Meeker and Meeker (1973) found that when tests were translated, Hispanic students tended to do better on the Spanish versions than on the English versions.

But on the other hand, Anastasi and Cordova (1953) and Manuel and Wright (1929) found no significant differences in performance on the Spanish and English versions of achievement tests. Clearly a factor sometimes overlooked in these studies is the degree of native language proficiency possessed by the students as compared to their command of English.

Some researchers (Oller and Perkins, 1978) have even hypothesized that intelligence, achievement, and personality instruments basically test language proficiency. These authors have presented results of factory analytic studies in support of their claim. Given the inevitable dependence of achievement and intelligence tests on language, it seems logical that such tests would have decreased validity and reliability for less than proficient English speakers. Concerning the effects of language on the generalizability of mathematics achievement scores for bilingual students, Llabre and Cuevas (1981) found that scores from mathematics achievement tests (concepts and applications) for students whose first language is Spanish are less reliable than what is usually reported in the literature as acceptable. Given the results of their research, Llabre and Cuevas (1981) also report that the primary language of instruction (English or Spanish), the level of reading proficiency in the language of instruction and the skill being measured should be taken into account when interpreting mathematic achievement test scores for linguistic minority students.

The previous discussion dealt with language in general as it relates to the learning and assessment of mathematics. Concerning specific language areas, reading related skills appear to play a significant part in certain types of mathematics achievement. A composite of results obtained by Banting (1922); Roling, Blume and Morehart (1924), and Stevenson (1925) includes the following reading related causes of failure in solving arithmetic word problems: (1) difficulties with word recognition; (2) lack of knowledge of vocabulary; (3) careless reading of the problem; and (4) lack of understanding the quantitative relationships. More recently, Hargis and Knight (1971) report that while reading materials have been developed which match the child's language, mathematics materials often present a mismatch between the vocabulary of the text and the student's vocabulary. It is Hargis and Knight's contention that more attention must be paid to reading related skills since all mathematics ideas are "imbedded in definite language statements with clauses signaling relationships and nouns signaling ideas." In addition to reading skills, Cathcart (1974) reports that the student's listening ability in the classroom has a significant effect on his/her mathematics achievement.

The object of the preceding discussion was to place an emphasis on language as it relates to the learning of mathematics since proficiency in the language of instruction is so important. The child who comes from a home where English is the main language spoken will have heard many of the language constructions used in the mathematics classroom. One cannot make the same assumption for the child whose native language is not English. Research efforts must be directed towards the analysis of language skills and language-related teaching strategies which will facilitate the language minority student's learning of mathematics.

Future Directions

Throughout the search of the literature one glaring fact is evident--there is a dearth of research concerning the nature of the relationship between language (in general) and mathematics learning and the role language plays in the assessment of mathematical concepts and skills. When the focus is placed on language minority students, the absence of research studies is extremely apparent. Based on the premises that language does play an important part in mathematics learning and that language factors account, in part, for the language minority student's underachievement in mathematics, future research efforts should be directed toward multivariate studies of the relationships among selected aspects of mathematics and various language variables. Careful consideration must be taken in incorporating levels of student language proficiency, socioeconomic status, age and sex variables as factors in the research designs.

With respect to language effects on the assessment of mathematics achievement, studies need to be conducted which examine the relationship between the level of reading difficulty of the test and student's performance. As with all research conducted with limited English proficient students, the definition and assessment of language proficiency must be carefully thought and carried out.

More specifically, the following are some of the questions which are suggested from the above two areas of concern:

1. What is the nature of the reading and language (general) difficulties present in selected areas of mathematics (concepts, problem solving, etc.)?

2. Is there a language effect upon achievement in computation?

3. To what extent do mathematical skills transfer from one language to another?

4. What is the relationship of reading achievement in the student's home language and performance in problem solving in mathematics for bilingual and limited English proficient students?

5. To what extent do specialized language methodologies, such as English as a Second Language (ESL) in the content areas, facilitate learning in mathematics?

These, then, are just a few of the many challenging questions for study. Their answers will hopefully provide equity of educational opportunities in mathematics for the language minority students.

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Women and Mathematics: State of the Art Review*

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Equity in Mathematics Core Conference
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In order for me to write a "state of the art" review about anything, I have to believe that knowledge exists which is socially significant, educationally important, and based on firm scholarship. Without any one of these three, it would be a waste of time for me to write, and anyone else to read, such a paper. I firmly believe that there is knowledge about women and mathematics that meets these criteria.

The problem of women and mathematics is socially significant and educationally important. If one is committed to the belief that women should have the same opportunity as do men to participate in all aspects of society, then the issue of sex-related differences in mathematics becomes vital. Without adequate preparation in mathematics, people are effectively filtered out of most post secondary education options and an increasing number of jobs and professions. The option of career advancement or change in adulthood is also severely handicapped by a lack of mathematical training. While inadequate training in mathematics hampers everyone, many more females than males fail to achieve their full potential in mathematics. This appears to me to be one of the most serious inequities in education that currently exists. Without mathematical knowledge and skills, women will never be able to achieve equity in society.

There is a firm knowledge base about women and mathematics. There are a great many data from a large variety of sources about enrollment patterns in high schools and universities. Data about achievement by females and males are available from national, state, and local assessments. Sex has been included as a variable in mathematics education research for decades and since about 1974, studies have been done which have increased significantly the knowledge of factors related to sex differences in mathematics. In addition, many intervention programs have been designed and implemented. These programs, while in most cases not well evaluated, have increased the intuitive knowledge that many have about the extent of the problem and procedures which might be effective in its solution.

I am not going to review all of these data and studies in this paper. There are available other reviews which do that job (Fox, 1980; Lantz, 1980). Instead, I would like to share with you my synthesis of what the problem is, what causes it, and some possible solutions. Hopefully, the synthesis is based on the knowledge I have gained by studying the problem intensively for about a decade and through my contacts with many others involved in the area. I would like to caution you, however, that I plan to raise serious questions about many ideas which permeate the rhetoric about women and mathematics. I am deeply committed to achieving equity in mathematics education for females, as well as all other groups. However, I do not believe that equity can be achieved unless we base decisions for action on firmly established knowledge.

What is the Problem?

There are three components of the problem: participation in mathematics courses in secondary and post secondary years, participation in adult mathematics-related occupations, and achievement or performance in mathematics. While there have been sex-related differences in the percentages of females and males who are enrolled in mathematics classes in secondary schools, the best information we have on a nationwide basis is that these differences are not dramatic until post secondary education.

Consider the information collected in two national surveys and reported recently by Armstrong (1981). While these data always show that more

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Insert Table 1 about here.

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males than females elect mathematics courses, with the differences increasing at more advanced levels, such differences do not appear dramatic - nor do they explain the extreme differences found in participation in mathematics related careers.

Instead of data collected from a nationwide sample, consider data collected in a more restricted area - the State of Wyoming (1978). Indeed strong differences are evident in mathematical preparedness between girls and boys.

Insert Table 2 about here.

What happens when we look at individual schools rather than compiling across the nation or a state? Wide variation in enrollment patterns exist, with more females than males actually enrolled in advanced math classes in some schools. I am convinced that while enrollment trends may be encouraging on a broad scale, it is only by looking at individual schools that meaningful assessment of females' enrollment in advanced math courses can be made.

All the data available indicate that the differential enrollment is greatest in post-secondary mathematics courses and in all programs where mathematics is a key component (such as Engineering or Computer Science). By adulthood, the vast majority of people who use mathematics are male.

Differences in Achievement

Before 1974, most studies reported male superiority in mathematics learning (Fennema, 1978). These studies usually used random samples of females and males in secondary schools. Since traditionally, females have not chosen to study mathematics as often as have males in advanced secondary school classes, a population of males who had spent more time studying mathematics was being compared to a population of females who had studied less mathematics. Since the single most important influence on learning mathematics is studying mathematics, it would indeed be strange if males had not scored higher on mathematics achievement than did females.

During the 1974-78 years, there were a number of studies published which indicated that sex differences in favor of males were not as strong as had been believed previously. I hypothesized at about this time that sex differences in mathematics might be eliminated if schools would somehow ensure that girls elect to study mathematics as often as boys did. However, within the last 2-3 years, a number of studies have been reported which have made me at least partially reject that hypothesis. These studies have carefully controlled the number of mathematics courses studied by both girls and boys and have also used items of differing cognitive complexity to assess learning.

The California State Assessment of Mathematics was done in 1978 (Student Achievement in California Schools, 1978). Students in Grades 6 and 12 (12- and 18-year-olds) who reported studying the same number of mathematics courses, were tested on a variety of content areas with item of differing cognitive levels. A committee was named to evaluate the results and concluded that girls outachieved boys in computational or lower cognitive level tasks while boys tended to score higher on higher cognitive level tasks. Armstrong (1980) collected data from a sample of students from the entire U.S. and concluded, "Twelfth grade males scored significantly higher than females on the problem-solving subtest. Thirteen-year-old females scored significantly higher on the lower level mathematical skill of computation." The Mathematics Assessment of the Second National Assessment of Educational Progress indicated also that females were somewhat better in computational tasks than were males, while males outachieved females in higher level cognitive tasks (Fennema and Carpenter, 1981). Just how strong are the reported differences? In a meta analysis of studies dealing with quantitative ability (which from an inspection of the studies used can be roughly equated with mathematics achievement), Hyde (1981) concluded that gender differences, while statistically significant, are in actuality quite small. Sex differences accounted for little of the variability in overall performance on the tests used in the studies she reported.

One can only conclude from a variety of sources that while sex-related differences in achievement are not always found, when they are found, they indicate that boys outachieve girls in high level cognitive tasks. Differences in both course enrollment and achievement are very school specific. In some schools, no differences are found, while in others, differences are consistently found.

Why Differences Exist

What is the cause of these sex-related differences in mathematics, both election to study, and achievement? Involved is the cognitive acquisition of mathematics by females, as well as the attitudes or affective beliefs held by females, male peers, parents, and educators toward females as learners of mathematics. The cognitive and affective components are so intertwined that it is difficult if not impossible to separate them. Not only are they intertwined, but they are developed over a period of years in a complex social matrix which involves home, community, and school. In addition to these social and educational influences, there has been increasing discussion in the last few years about the influences of genetic variables. It is outside the scope of this paper to discuss the arguments about genetic influences (See Stage, 1981; Fennema, 1981 for a more thorough discussion). However, the knowledge base from which conclusions about genetic differences are drawn is very small and conclusions are usually based on poorly interpreted data. In addition, the genetic argument, even if it were true, could never explain the large differences found in participation in mathematics-related careers. Therefore, it is more sensible to focus on variables which are amenable to change.

One cognitive variable which many believe might help in understanding sex-related differences is spatial visualization, or the ability to manipulate rigid figures mentally. Even though the existence of many sex-related differences is currently being challenged, the evidence is still

persuasive that in many cultures, male superiority on tasks that require spatial visualization is evident beginning during adolescence (Fennema, 1975; Maccoby and Jacklin, 1974). The relationship between mathematics and spatial visualization is logically evident. In mathematical terms, spatial visualization requires rotation, reflection, or translation of rigid figures. These are important ideas in geometry. Many mathematicians believe that all of mathematical thought involves geometrical ideas (Bronowski, 1947). Therefore, if spatial visualization items are geometrical in character and if mathematical thought involves geometrical ideas, spatial visualization and mathematics are inseparably intertwined.

Not only are spatial visualization skills related to ideas within the structure of mathematics, but spatial representations are being increasingly included in the teaching of mathematics. For example, the Piagetian conservation tasks which are becoming part of many school programs, involve focusing on correct spatial attributes before quantity, length, and volume are conserved. Most concrete and pictorial representations of arithmetical, geometrical, and algebraic ideas appear to be heavily reliant on spatial attributes. The number line, which is used extensively to represent whole numbers and operations on them, is a spatial representation. Illustrating the commutativity of multiplication by turning an array 90 degrees, involves a direct spatial visualization skill.

Although the relation between the content of mathematics, instruction in mathematics, and spatial visualization skills appears logical, results from empirical studies which have explored the relationship are not consistent. Such studies are usually correlational, which gives little information on the impact spatial visualization has on learning mathematics. The hypothesis that I and my colleagues are currently investigating is that the critical relationship between mathematics and spatial visualization is not direct, but quite indirect. It involves the translation of words and/or mathematical symbols into a form where spatial visualization skills can be utilized.

We know that females tend to score lower on spatial visualization tests than do males. What we do not know is if females differ from males in their ability to visualize mathematics. Also not known is if good spatial visualizers are better at this translation than are poor spatial visualizers. However, I am increasingly convinced that there is no direct causal relationship between spatial visualization skills and the learning of mathematics in a broad, general sense. While I am continuing to investigate the impact of spatial visualization skills, I am less convinced than I was that spatial visualization is important in helping understand sex-related differences in the studying and learning of mathematics.

Classrooms don't appear to use mathematical representations which either encourage or require the use of spatial visualization skills. While some primary mathematics programs encourage the use of concrete and pictorial representations of mathematical ideas, by the time children are 10 to 11 years old, symbolic representations are used almost exclusively. Perhaps boys, more than girls, utilize the concrete representations during primary years, and thus, develop higher skills in using spatial visualization in learning mathematics. As far as I know, however, no one is investigating a hypothesis.

Affective Variables

If sex-related differences in mathematics can't be explained by cognitive variables, are there other variables which will help? Several variables that I label as affective provide important insight into why females elect not to study mathematics beyond minimal requirements and are not learning math as well as are males.

Affective variables have to do with feelings, beliefs, and attitudes. The affective domain is a complicated one and has received less attention than the cognitive domain, partly because variables within this domain are difficult to define, measure, and understand. All too often, all affective variables have been lumped together into one large conglomerate and labeled as attitudes. However, this type of combining often masks many important things.

There has been an increasing amount of literature published which deals with specific affective variables and their relationship to sex-related differences in mathematics study (Fox et al., 1979; Fennema, 1978; Reyes, 1980). Two well-defined variables (confidence and perceptions of usefulness) are closely related to studying mathematics, and one other complex variable (causal attributions) has been hypothesized to be an important determinant of electing to study mathematics (Wolleat et al., 1980).

Confidence in learning mathematics is related to self-esteem in general. High confidence in mathematics appears to be located at one end of a continuum and anxiety toward learning mathematics at the other end. Confidence in mathematics is a belief that one has the ability to learn new mathematics and to perform well on mathematical tasks. It often is measured by Likert-type scales which include items such as: I am sure that I can learn mathematics; I can get good grades in math; or I'm no good in math.

The literature strongly supports the fact that there are sex-related differences in the confidence-anxiety dimension. It appears reasonable to believe that lesser confidence, or greater anxiety on the part of females is an important variable which helps explain sex-related differences in mathematics studying.

While evidence exists in abundance that there are sex-related differences in this confidence/anxiety dimension related to mathematics, much is unknown about its true effect or how such feelings are developed. The relationship between spatial-visual processes and the confidence/anxiety dimension has not been explored. What effect do feelings of confidence have on cognitive processes involved in learning mathematics and in solving mathematical problems and vice versa? Are feelings of confidence stable within individuals across time and across a variety of mathematics activities? Does lessening anxiety increase either learning or the willingness to elect to study mathematics? Do low levels of confidence affect females differently than they do males? Are there really sex differences in confidence toward mathematics, or (as many have hypothesized, Nash, 1979) are females just more willing to admit their feelings than are males?

Currently, there are many studies underway which will help in answering these questions. Until they are available though, one must just accept the evidence that females, across a wide age range, do report more anxiety and less confidence toward mathematics than do males.

Even when females succeed in mathematics, they attribute their success to factors other than their own ability, such as luck, much more than do males (Wolleat et al., 1980). Females' causal attribution patterns are different from those of males and causal attribution theory holds promise in helping understand sex-related differences in mathematics. Causal attribution has to do with what one believes causes successes and failures. In one model (Weiner, 1974), attributions of causes of success and failure are categorized into a 2 x 2 matrix with locus of control (internal-external) being one dimension, and stability and instability the other.

	Internal	External
Stable	Ability	Task
Unstable	Effort	Luck/ Environment

Figure 5

One can believe that success or failure occurs in mathematics because one is smart or dumb (ability), one tried or did not try (effort), the mathematics one is doing is easy or difficult (task), or one has or doesn't have a good teacher (luck/environment).

Pattern of causal attributions affect persistence in achievement oriented behavior. In a somewhat simplistic summary, if one attributes success to an internal, stable dimension (ability), then one expects success in the future and will continue to strive in that area. If one attributes success to an unstable or an external cause (e.g, the teacher), then one will not be as confident of success in the future and will be less apt to strive or persist. A somewhat different situation is true of failure attributions. If one attributes failure to unstable causes such as effort, one might work harder the next time and failure could be avoided. With this situation, the tendency to approach or persist at tasks will be encouraged. Attribution of failure to a stable cause, on the other hand, will lead one to believe that failure can't be avoided.

While we must be careful of overgeneralizing data and concluding that all males behave one way and all females another way, many studies have reported that females and males tend to exhibit different attributional patterns (Deaux, 1976; Bar-Tal & Frieze, 1977). Males tend to attribute successes to internal causes, and failures to external or unstable causes. Females tend to attribute successes to external or unstable causes and failures to internal causes. This attributional pattern has been observed in mathematics (Wolleat et al., 1980) as well as in other areas and probably affects both long term persistence (election of courses) and short term persistence (sticking with a hard problem). This particular combination of attributions (success attributed externally and failure attributed internally) has been hypothesized to strongly affect academic achievement and, in particular, females' achievement.

Another effective variable which helps explain females not electing to take mathematics is the perceived usefulness to them of mathematics (Fox, 1980; Fennema, 1978). Mathematics is a difficult subject and not particularly enjoyable for many learners. Why should one study it if it is of no future use? Females in secondary schools as a group indicate that they do not feel they will use mathematics in the future. Females, more than males, respond negatively to such items as: I'll need mathematics for my future work or mathematics is a worthwhile and useful subject. Males, as a group, are much more apt to report that mathematics is essential for whatever career they plan. As early as 6th grade, these sex differences appear. If females do not see mathematics-related careers as possibilities, they will also not see mathematics as useful.

In addition to indicating more negative beliefs than do males on these specific affective variables, females also report that they perceive that parents, teachers, and counselors are not positive toward them as learners of mathematics. In addition, males, more than females, starting at least as early as grade 6, stereotype mathematics as a male domain at much higher levels than do females.

Society's stereotyping of math as a male domain, is at least a partial cause of girls' less positive attitudes toward mathematics. While females deny that math is a male domain, everything else in society is at variance with this denial. Of course, the main users and teachers of mathematics are male. Male peers, much more than females, say that math is a male domain. Evidence also exists in abundance (Fox, 1980) that parents, teachers, and counselors believe that mathematics is a more appropriate activity for males than it is for females. These beliefs are undoubtedly communicated to girls in a variety of subtle and not so subtle ways. Math is perceived to be inappropriate for girls. It seems logical to believe that when young girls feel mathematics is inappropriate, they will feel anxious about succeeding in it and have more negative attitudes because they must, at least partially, deny their femininity in order to achieve in mathematics.

While the evidence is strong that there are sex-related differences in confidence in mathematics, perceived usefulness of mathematics, and causal attributions of success and failure in mathematics, I'm always somewhat puzzled as to what implications such knowledge has for planning change. I have no trouble saying and believing that females should learn mathematics to the limits of their ability, that if they were more confident, they would learn mathematics better, and that if they perceived mathematics as more useful or attributed success and failure differently, they would persist in studying mathematics. Certainly just having that knowledge gives me only limited power to promote change. Changing beliefs and actions by learners, while not impossible, is difficult. However, educational systems can promote such change, and an understanding of how schools influence females' beliefs about mathematics is helpful.

Educational Variables

While the entire social milieu influences how well one learns as well as how one feels about mathematics, the most important influences occur within the classroom where mathematics is taught. Learning environments for girls and boys within classrooms, while appearing to be the same, differ a great deal. The most important component of the learning environment

is the teacher. Part of the teacher's influence is in the learner's development of sex role standards. These sex role standards include definitions of acceptable achievement in the various subjects. The differential standards for mathematics achievement is communicated to boys and girls through differential treatment as well as differential expectations of success. To start with, teachers interact more with boys than they do with girls. Boys generally receive more criticism for their behavior than do girls and boys also receive more praise and positive feedback than do girls. Boys just seem to be more salient in the teacher's view than do girls.

Many people feel that differential treatment of girls and boys is a result partially of differential teacher expectation of success or failure by girls and boys. The relevant discussion goes something like this. Because of societal beliefs that males are better at mathematics than are females, teachers expect that boys will understand high cognitive level mathematics better and girls will do better on low level mathematics tasks like computation. This belief is communicated in a variety of subtle and not so subtle ways to both boys and girls. For example, a teacher might encourage boys more than girls to stick with hard mathematical tasks until solutions are found. The teacher, with good motivation about preventing failure, assists girls more than boys to find the solution to hard problems. Teachers might call on boys more often to respond to high level cognitive questions and call on girls more often to respond to low level tasks. If this type of behavior occurs, boys and girls could intuit that boys were better at high level cognitive tasks and girls were better at low level cognitive tasks. Not only could students conclude that high level tasks are easier for boys, they could also conclude that such mathematics was more important for boys since teachers encouraged boys more than girls to succeed in such tasks. In addition to these subtle messages, boys would actually be practicing high level cognitive tasks more than would girls. Since students learn what they practice, boys would learn to do the problem-solving activities better than would girls.

The hypothesis of differential teacher expectation is intuitively logical and indeed, Brophy and Good (1974) report that teachers' expectancies are related to the way they interact with students. An interesting study by Becker (1981) which was done in 10th grade geometry classes, confirms this also. Becker hypothesizes that the sex-related differences she found in teacher/student interactions were strongly related to differential teacher expectancies. However, other studies designed explicitly to examine teacher expectancies report no differences in expectancy of success in mathematics by teachers for girls and boys (Parsons et al., 1979) and students report that teachers have higher expectancies of success for girls than for boys (Fennema et al., 1980). Once again, no clear cut conclusion about teacher expectancies of girls and boys in learning mathematics can be reached.

The problem of differential treatment of male and female students by teachers is well documented and I have no doubt that it strongly influences learning. It is easy to conclude that, but the longer I study the problem, the more complex it becomes. Most overt behavior by teachers appears to be nonsexist and fair to most students. In many cases, teachers interact more with boys because they feel they must to maintain control. Many negative interactions occur between boys and teachers. On the surface,

teachers' interaction with girls are more positive and what I have always considered to be good educational practice. However, the end result appears to be negative. I believe that at least a partial result is that females, more so than males, are not reaching one of the important goals of mathematics education, that of becoming thinkers who are independent problem solvers and who do well in high level cognitive tasks. Girls, much more than boys, fail to become autonomous in mathematics. This is indicated by girls' more negative attitudes related to the ability to perform high level cognitive tasks, specifically, confidence in learning mathematics and attributional patterns which indicates lack of control in mathematics outcomes as well as lowered performance in problem-solving tasks. In order to become increasingly autonomous, one must develop confidence in one's ability to do difficult learning tasks and also believe that one is in control of the outcomes of achievement striving.

Not too much is known about how one becomes an autonomous learner. It is believed that dependent/independent behaviors are developed by the socialization process, mainly within social interactions. Young girls, more so than boys, are encouraged to be dependent. Girls receive more protection and less pressure for establishing themselves as individuals separate from parents. Because of this, girls are less likely to engage in independent exploration of their worlds (Hoffman, 1975). Because of the sex-typed social reinforcement of dependent/independent behaviors, children enter school with girls tending to be more dependent on others and boys tending to be more self-reliant. What appears to happen is that schools merely reinforce and further develop in girls and boys the dependent/independent behaviors they bring to school. This set of behaviors is particularly apparent in mathematics.

What prohibits girls, more than boys, from becoming autonomous learners of mathematics? It would be nice if an answer to this question could be written which would be both accurate and easily understood, but that is not possible. The factors that cause behavior are many, varied, and interact in a complex way. Indeed, there probably are as many combinations of causative factors as there are individuals. Many influences on development of behaviors are subtle and difficult to identify. However, I firmly believe that a large component of development of autonomous learning behaviors in mathematics takes place in mathematics classrooms.

Changing Schools

Can schools be changed so that females learn better and elect to study more mathematics? All too often, comments are addressed to me that imply that schools can't do much. The argument goes like this. Because mathematics is stereotyped male, and because stereotyping of sex roles is so deeply embedded in society, schools are powerless to improve females' studying of mathematics until society changes. Let me say as loudly and emphatically as I can that that argument is fallacious. Schools can increase females' studying and learning of mathematics. Let me cite some evidence that shows strongly that schools can be effective. I want to talk specifically about two intervention programs in the U.S. that have been intensively evaluated. The first program, called MULTIPLYING OPTIONS AND SUBTRACTING BIAS is aimed specifically at increasing females' belief of the usefulness of math (Fennema et al., 1981). The rationale behind this program is that merely telling high school females about the importance of mathematics is insufficient. Forces which influence these girls

to make their decisions are complex and deeply embedded in societal beliefs about the roles of males and females. Asking adolescent girls to change their behavior without changing the forces operating upon them would place a very heavy burden on their shoulders. What should be done is to change the educational environment of these females so that they can recognize the importance of mathematics. MULTIPLYING OPTIONS AND SUBTRACTING BIAS was designed to change four significant groups' beliefs and behavior about women and mathematics.

MULTIPLYING OPTIONS AND SUBTRACTING BIAS is composed of four workshops: one each for students, teachers, counselors, and parents. Each of the four workshops is built around a unique version of a videotape designed explicitly for the target audience. Narrated by Marlo Thomas, the tapes use a variety of formats, candid interviews, dramatic vignettes, and expert testimony to describe the problem of mathematics avoidance for the target audience. The videotapes and accompanying workshop activities make the target audiences aware of the stereotyping of mathematics as a male domain which currently exists, females' feelings of confidence toward mathematics, the usefulness of mathematics for all people, and differential treatment of females as learners of mathematics. Discussed specifically are plans for action by each group. The workshops, each of which is about two hours long, are designed to impact on a total school.

An extensive evaluation of this program (Fennema et al., 1981) indicated that the MULTIPLYING OPTIONS AND SUBTRACTING BIAS series can substantially influence students' attitudes about mathematics, the stereotyping of mathematics and their willingness to take more mathematics courses.

The other intervention program is one developed, planned, and implemented by the San Francisco Bay Area Network for Women in Science (now called the Math/Science Network). The Network is a unique cooperative effort undertaken by scientists, mathematicians, technicians, and educators from 20 colleges and universities, 15 school districts, and a number of corporations, government agencies, and foundations. The goal of the Network is to increase young women's participation in mathematical studies and to motivate them to enter careers in science and technology. While the evaluation was much more extensive than I am reporting, I would like to report to you the evaluation of seven conferences which took place in the spring of 1977 and 1978 (Perl & Cronkite, 1979). Supported by the Women's Educational Equity Act of the federal government, these conferences were designed to increase the entry of women into mathematics/science-oriented careers.

These one-day conferences involved bringing junior and senior high school girls together in a central location. They were presented with a general session with a panel or main speaker, one or two hands-on science/math workshops, and one or more career workshops which provided opportunities for interaction with women working in math/science-related fields. Subjects of this evaluation were 2,215 females who volunteered to attend the conferences. Pre- and post-conference questionnaires were administered and responses analyzed. The evaluators concluded that "the conference (1) increased participants' exposure to women in a variety of technical and scientific fields, (2) increased participants' awareness of the importance of taking mathematics and science-related courses, and (3) increased participants' plans to take more than two years of high school mathematics."

The two intervention programs described indicate quite clearly that it is possible to change females' mathematics behavior, and to do so in relatively short periods of time.

Conclusion

Let me start the concluding remarks with some unproven statements that I often hear.

1. Females prefer to learn their mathematics in classroom discussions. Males prefer to work individually.
2. Classroom interactions are more important to females than they are to males.
3. Females are passive; males are active.
4. Female teachers teach mathematics more poorly than do male teachers.

These conclusions come from intuitive belief, isolated studies, and/or a poorly done or interpreted piece of research (see Fennema, 1981, for an expanded discussion).

Statements such as these are causing me major concern. Am I, and others who are deeply concerned with helping women achieve equity helping females to achieve true equity in mathematics education? Or are we helping to perpetuate the myth that there are large and nonchangeable sex-related differences in mathematics? Are we, indeed, creating a new mythology of female inadequacy in the learning of mathematics?

There are some statements which I believe are based on sound evidence.

1. There are still sex-related differences in electing to study mathematics in high school. While not as dramatic as were once suggested, females tend not to study, as much as do males, the most advanced mathematics courses and courses peripheral to math, such as computer science, statistics, and physics. It appears that the size of the differences varies tremendously by school and by region of country. At the post high school levels, differences are still large.
2. Even when amount of mathematics studied is controlled, females appear not to be learning math as well as are males in some instances. The trend that should be of concern to us all. When females excel, it is in lower level cognitive tasks. Even when females and males report they have been enrolled in the same mathematics courses, males perform better on more difficult and complex tasks.
3. There are psychological variables which may help in understanding sex-related differences. Females, as a group, more than males, as a group, have less confidence in learning mathematics, perceive mathematics to be less useful to them, and attribute successes and failures in mathematics differently.
4. The classroom learning environments are different for females and males in a variety of ways.
5. Males perform better than females on tests of spatial visualization although the impact of spatial visualization on the learning of mathematics is largely unknown.
6. Within schools lies a major portion of causation of sex-related differences in mathematics.

Table 1

National Sex Differences in Participation

in High School Mathematics Courses[a]

	NAEP[c]		WIM[d]	
	Females	Males	Females	Males
Algebra I	74	71	79	81
Geometry	51	52	55	59
Algebra II	36	38	42	54
Trigonometry	17	21	27	31
Probability/Statistics	8	10	5	10
Computer Programming	8	10	13	18
Precalculus	b	-	18	22
Calculus	4	6	7	8

[a] From Armstrong, J. M. Achievement and Participation of Women in Mathematics: Results of Two National Surveys. Journal for Research in Mathematics Education, 12, 5, November 1981, 356-373.

[b] Precalculus and Calculus combined.

[c] Collected by self report of 17 year olds.

[d] Collected by self report of high school seniors.

Table 2

Percent of College-Bound Seniors Attaining

Each of Sex Levels of Mathematical Preparedness*

	Percent	
Level	Female	Male
1**	15	30
2	21	36
3	54	64
4	71	78
5	93	94
6	100	100

*Kansky, B., & Olson, M. Mathematical Preparation Versus Career Aspirations: Study of Wyoming 1978 High School Seniors (The Science and Mathematics Teaching Center, Box 3992 University Station, Laramie, Wyoming 82071), p. 13.

**Level 1 is the highest level of mathematical preparedness.

From Fennema, E. The Sex Factors in Fennema, E. (Ed). Mathematics Education Research: Implications for the 80's. Association for Supervision and Curriculum Development 1981, pp. 92-110.

This last statement, while sounding unduly harsh to many, is the most positive thing I have said in this paper, and indeed makes the problem of females and mathematics solvable. Schools can influence what happens. Schools do make a difference. Schools can solve the problem of females and mathematics.

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Blacks in Mathematics:

The State of the Art

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Blacks in Mathematics: The State of the Art

During the seventies much attention was focused on the status of blacks in relation to mathematics and the mathematical sciences. A calamitous set of interrelated occurances served as catalyst for this long overdue attention. The black population of the United States was approximately 11.6% of the total population in 1976 yet less than 5% of all BS degrees in Mathematics was awarded to blacks; less than 3% of all BS degrees in Engineering was awarded to blacks (National Science Foundation, 1980(a)). Black high school students were enrolling in mathematics courses at an alarmingly low rate and were continuing to perform at a level lower than many other groups of students as measured by standardized tests. These facts, among others, dictated that a serious look at the mathematical education of black youth was needed.

The participation of blacks in mathematics courses and the mathematical achievement of blacks at the pre-college years are concerns that must receive national attention. Each concern represents highly complex, multidimensional issues having far reaching consequences. In this paper I will review efforts to document the various dimensions of the problem and report on attempts of researchers to find effective solutions.

Participation in Mathematics Courses

All available data on participation of high school students in mathematics courses indicate that blacks are enrolling in fewer mathematics courses than most other racial/ethnic subgroups. The National Longitudinal Study of the High School Class of 1972 (NLS-72) reported that the average number of semesters of mathematics taken from 10th to 12th grade was 3.4 and 3.8 for black and white students, respectively (Vetter, Babco, & McIntire, 1979). Almost 50% of the black students enrolled in mathematics for 0-2 semesters and less than 20% enrolled for six or more semesters. Table 1 includes participation rates of the high school class of 1972 by racial/ethinc group.

TABLE 1

Participation Rates in High School
Mathematics Courses by Racial/Ethnic Group

Racial/Ethnic group	Semesters of High School Mathematics 10th through 12th Grade				
	0-2	3-5	6 or more	total	average semesters
American Indian	63.3	25.9	10.9	100.0	2.6
Black	42.2	38.4	19.4	100.0	3.4
Hispanic	43.7	39.0	17.3	100.0	3.3
Asian American	23.1	29.4	47.6	100.0	4.6
White	36.7	33.3	30.0	100.0	3.8

Source: Adapted from Vetter, Betty M., Babco, Eleanor L., and McIntire, Judith E., Manpower Comments, November 1979, p. 20

The High School and Beyond Study, a national longitudinal study for the 1980's, reported that substantial racial differences still exist in the number of mathematics and science courses taken by high school seniors (Peng, Fetters & Kolstad, 1981). In 1980, only 35% of all black seniors reported having taken three years or more of mathematics. Only 19% of all black seniors took three years or more of science (see Table 2).

TABLE 2

Percentage of 1980 High School Seniors
Taking Three Years of More of Coursework
in Mathematics and Science, by Racial/Ethnic Group

Racial/Ethnic Group	Mathematics	Science
American Indian/ Alaskan Native	22	12
Black	35	19
Hispanic	27	14
Asian American	48	32
White	33	23

Source: National Center for Educational Statistics, High School and Beyond, 1981, p. 2.

Although the data in Table 2 reveal a very low participation rate in mathematics courses by blacks, the larger picture indicates that between 1977 and 1980 the percentages of black students taking specific mathematics courses such as Algebra I, Algebra II, and Geometry actually increased. The 1977-78 assessment of the National Assessment of Educational Progress indicated that among 17-year-olds the percentage of black and white students who had taken Algebra I was 55% and 75%, respectively. The percentage of 17-year-old black and white students who had taken Algebra II was 24% and 38%, and for those who had taken Geometry, 31% and 55% respectively. These data are reported in Table 3 (NAEP, 1979). In the High School and Beyond Study, the percentage of black and white high school seniors who had taken Algebra II was 39% and 50%, and for those who had taken Geometry, 38% and 60%. Percentages for all subgroups in the class of 1980 are given in Table 4.

Table 3

Percent of 17-Year-Olds
Who Had Taken Algebra I,
Algebra II, and Geometry in 1977-78, by Race

	Race	
Mathematics Course	Black	White
Algebra I	55	75
Algebra II	24	39
Geometry	31	55

Source: National Assessment of Educational Progress, Mathematical Knowledge and Skills, p. 45.

TABLE 4

Percentage of 1980 High School Seniors
Taking Mathematics and Science Courses
by Course Title and Racial/Ethnic Group

Mathematics Course	Racial/Ethnic Group				
	American Indian	Black	Hispanic	Asian	White
Algebra I	61	68	67	88	81
Algebra II	32	39	38	76	50
Geometry	34	38	39	79	60
Trigonometry	17	15	15	50	27
Calculus	5	5	4	22	8
Physics	17	19	15	35	20
Chemistry	24	28	26	59	39

Source: National Center for Educational Statistics, High School and Beyond, 1981, p. 5.

The consequences of taking so few mathematics courses in high school are severe. Sells (1978) reported that career choices, in particular careers in the mathematical, physical and biological sciences, are seriously limited without four years of secondary school mathematics, including Algebra I, Algebra II, Geometry, and at least one year of pre-calculus mathematics. The NLS-72 reported that a positive correlation existed between the number of semesters of mathematics completed in high school and the completion of the bachelor's degree on schedule (in the fall of 1976). Further, students with more mathematics and science coursework in high school were more likely to choose science and other quantitative fields in college, both black and white (Peng, Fetters, & Kolstad, 1981, 40).

The low black enrollment in mathematics is also seen in post-secondary study. Few black students are majoring in the mathematical sciences in college. Obviously, enrollment in high school mathematics courses impacts on this in a major way. In 1975-76, blacks earned 2.9% of the 45.6 thousand BS degrees awarded in engineering, 5.8% (out of 5.6 thousand) of the BS degrees in computer science, 4.9% (15.9) in mathematics, and 2.9% (21.2) in physical science (National Science Foundation, 1980 (a)). The proportion of blacks in the U.S. population in 1977 was 11.6%, hence, the above data show the serious underrepresentation of blacks in highly quantitative areas of study. There is little reason to expect that these trends have changed significantly in the last five years or that it will change significantly in the near future. Only 48% of the black students in the High School and Beyond sample expect to pursue a 4-year college degree. Although the career choices of blacks were not identified, the percentage of the total sample planning to enroll in various scientific fields were as follows: engeneering - 10%, computer and information sciences - 5%, biological sciences - 5%, physical sciences - 2%, mathematics - 1% (Peng, Fetters, & Kolstad, 1981).

Mathematical Achievement

The mathematical achievement of black students, as measured by the National Assessment of Educational Progress, continues to lag behind the achievement of white students in every test category and at each age level: 9, 13, and 17 (NAEP, 1980). Table 5 contains data showing the mean performance percentages of black students across all categories tested in the 1977-78 national assessment. Although these data are sobering to say the least, they represent a significant positive change from the 1972-73 national assessment. Nine-year-old black students "showed significant increases in performance

TABLE 5

Mean Performance Percentages
of Black 9, 13, and 17-Year-Olds on 1977-78
National Assessment of Educational Progress

Title of Set of Mathematics Exercise	Age 9 Mean Percentage		Age 13 Mean Percentage		Age 17 Mean Percentage	
	Nation	Black	Nation	Black	Nation	Black
Mathematical Knowledge	65.94	54.92	66.87	52.82	71.71	56.24
Numbers & Numeration	67.45	55.41	71.20	56.75	77.90	63.03
Place Value	63.43	50.19	70.86	50.86	-	-
Basic Facts	76.90	64.08	90.80	80.60	93.74	86.79
Ordering	66.19	60.03	54.04	40.72	64.48	40.96
Number & Oper. Prop.	45.92	35.50	63.26	49.22	77.24	61.54
Geometry Knowledge	58.74	50.08	57.30	44.87	67.53	50.44
Meas. Knowledge	65.82	58.86	69.79	53.57	78.71	62.27
Mathematical Skills	43.34	32.55	51.88	35.10	58.97	41.39
Computation	35.44	26.65	51.68	35.26	67.17	49.60
Whole Number Comp.	52.36	40.12	82.67	69.49	87.54	79.48
Measurement Skills	52.91	41.60	54.75	33.53	56.72	33.26
Making Measurements	63.59	50.90	70.92	46.58	-	-
Reading Graphs & Tab.	58.91	43.73	68.85	52.22	72.56	55.96
Geometric Manipulation	44.44	33.63	45.81	33.41	54.76	35.87
Algebraic Manipulation	39.36	27.57	52.17	33.58	39.99	25.05
Solving Equation	44.52	34.32	63.74	43.93	35.20	17.95
Math Understanding	39.60	29.31	51.72	36.96	58.01	40.65
Math Applications	37.75	27.08	43.34	29.69	43.48	25.66
Routine Problems	36.66	24.03	41.43	26.90	41.55	23.03
One-step word Problems	44.34	20.10	52.08	33.72	57.09	35.38
Computation-Fractions			51.40	31.96	65.82	47.88
Computation-Decimals			54.70	32.95	75.22	54.72
Computation-Intergers			38.22	27.54	68.09	48.19
Finding Perimeter, Area and Volume			45.06	27.53	-	-

TABLE 5 (Continued)

Mean Performance Percentages
of Black 9, 13, and 17-Year-Olds on 1977-78
National Assessment of Educational Progress

Title of Set of Mathematics Exercise	Age 9 Mean Percentage		Age 13 Mean Percentage		Age 17 Mean Percentage	
	Nation	Black	Nation	Black	Nation	Black
Plotting Points			57.04	32.57		
Estimation			35.75	19.88	49.63	30.12
Multistep Word Prob.			36.82	24.17	35.10	15.93
Graph & Table Prob.			55.26	35.62	63.19	40.91
Geometric Problems			25.86	19.53	36.77	20.82
Comb, Stat, and Probab.			35.52	21.70	25.50	13.47
Nonroutine Problems					40.49	23.40
Measurement Problems					33.08	13.38

Source: National Assessment of Educational Progress, Mathematical Technical Report: Summary Volume.

and performance relative to the nation on the entire set of exercises and on the knowledge, skill and computation subsets of exercises. In addition, this group showed a significant increase in performance, relative to the nation, on mathematical application exercises" (NAEP, 1980, p. 12). Black 13-year-olds showed a significant increase in performance, relative to the nation, on mathematical skill exercises. Black 17-year-olds showed an increase on consumer problem exercises and no loss in performance in knowledge computation and reading graphs and tables, however, the mean performance of black 17-year-olds declined on many exercise sets with approximately 20 percentage points separating black and white test scores.

The NAEP findings suggest that serious problems exist in our mathematics classrooms for black students. An indepth study of the origin of these problems as well as the plight of learners of all ethnic/racial groups must be undertaken immediately. One recent study reported that more than a fourth of all 1980 high school seniors had taken remedial coursework in mathematics (Peng, Fetters, & Kolstad, 1981). Overall, high school students are performing poorly in mathematics!!

Contributing Factors

The above data on enrollment patterns and achievement of black students in mathematics confirm what has been recognized by black educators for years; black students and white students are not achieving at the same level. Naturally, the question of "Why?" is asked. At this time there are no definitive answers to this question. There are some beliefs and speculations about: a) factors that have a major impact on black students, and b) how black students learn in the current school environment.

Wilson(1978) has attempted to analyze the growth, development and education of the black child in today's society. According to Wilson, a black child's performance in a classroom reflects that child's socialization in a hostile, racist society. Wilson argues that schools exist to maintain and to perpetuate a cultural way of life. Most schools do not show sensitivity to black culture and hence it should not be surprising to find that the black child's academic performance "lag behind" that of whites in a school system designed to maintain white dominance.

Wilson states that black parents must take the major responsibility of socializing and educating black children in a way to increase achievement motivation and to decrease the black child's high need for dependency and low need for achievement. Such an emphasis would result in changes in the behavioral, motivational, and cognitive functioning of black children (Wilson, 1978).

Other factors commonly mentioned as reasons for lack of interest in taking mathematics by black students are: 1) lack of black role models, 2) lack of significant others, such as parents, who have interest in mathematical achievement, 3) lack of positive career counseling, 4) viewing mathematics as a subject for white males, 5) inability to see the usefulness and relevance of mathematics to their lives, both present and future, and of course, 6) lack of success in previous mathematics courses. These factors are all interrelated and have historical roots in centuries of institutionalized racism that perpetuated unequal education for black people. It should be pointed out that very little research has been conducted that attempted to ask black students why they did not take more mathematics. Much of what is "known" relative to why blacks do not take more mathematics is juxtaposed from the research on "women and mathematics." The effects of sex discrimination and mathematics participation and/or achievement have been extensively researched and reported (Jacobs, 1978). Similar studies of racial discrimination and its effects on mathematics achievement are not available at this time.

The effects of a lack of black role models in science and mathematics, including a lack of black mathematics teachers, on a student's choice of what mathematics to take in high school have not been sufficiently researched. Certainly, all participants in the educational process realize the importance of making students aware of the contributions of black scientists, both past and present, to the history of this nation. It seems reasonable that black students would seek to emulate black men and women who serve as strong role models, however, there is little research to support this contention.

Parents, relatives, and friends play a major role in a student's choice of mathematics or mathematics related career at the college level. Carey (1977) reported that 32% of the engineering students entering Purdue University in 1973 stated that their choice was heavily influenced by a relative, 26% said friends and high school courses, while only 14% said teachers and high school counselors. Careful research on the relative influence of parents, teachers, and counselors on the decision of black high school students to take advanced mathematics is in its embryonic stage.

The perceived usefulness of mathematics has often been stated as a factor influencing how much mathematics a student will take. Matthews (1980) reported that black females viewed advanced mathematics courses as essential when preparing for college entrance requirements but not useful to their future careers. Black males tended not to make this distinction.

Overall, few definitive reasons can be given for the acute underrepresentation of blacks in advanced mathematics classes. Equally elusive is an explanation for their consistent underachievement on mathematics tests. Research programs must begin immediately to address these issues, including a study of the learning environment of the mathematics classroom. Careful investigation of what teachers do, how they interact with black students, and the effects of this interaction on students' performance is desperately needed.

Intervention Studies

During the last two decades, research on minorities and mathematics education issues has been largely neglected (Ortiz-Franco, 1981). Even in 1982, only a small number of research studies aimed exclusively at the problems of blacks and mathematics can be identified.

A project designed to motivate black students toward careers involving mathematics is Blacks and Mathematics (BAM). BAM was funded by the Mathematics Association of America and has three major objectives:

1. to provide role models to encourage more black students to consider mathematics-based careers;
2. to influence counselors, teachers and parents to direct more black students into mathematics-based careers in which blacks are underrepresented;
3. to inform students, teachers, guidance personnel, and parents of the large number and variety of careers for which mathematics is a prerequisite.

BAM began its program during school year 1980-81. Results of this project are unavailable at this time.

Only four studies are found among the more than 90 projects funded by the National Science Foundation, Directorate of Science Education, Division of Science Education and Research, Fiscal Year 1980, that plan to investigate factors related to mathematics course-taking among blacks and minorities. These projects are listed below.

Permaul, Jane. Role Models for Adolescent Girls in Science and Mathematics. (SED 79-19023) Ending Date 02-28-81

Cohen, Arthur. Science Education for Women, Minority, and the Physically Handicapped Students in Community Colleges. (SED 79-20222) Ending Date 12-31-82

Scherrei, Rita. A Longitudinal Study of Women and Minorities in Science and Engineering. (SED 80-17651) Ending Date 05-31-82

LeBold, William. Improving Access and Guidance in Engineering: Research into Contributing Factors. (SED 79-19613) Ending Date 12-31-82.

The National Institute of Education, through its Learning and Development Unit has funded a small set of studies on minorities and mathematics education. Of the 19 projects funded, seven address issues pertaining to blacks or include minority students in their samples. The findings of the four projects listed below should add significantly to current knowldege in this area.

Gaston, Jerry. Factors Inhibiting Science Careers for Black Women. (NIE-G-78-0139)

Johnson, Robert. Psychosocial Factors Affecting the Mathematical Orientation of Black Americans. (NIE-G-79-0093)

Marrett, Cora. Minority Female Involvement in High School Science and Mathematics. (NIE-G-79-0110)

Morse, Linda. Relationship of Significant Others, Parental and Teacher Influences to the Development of Self-Concept, Science Attitudes and Achievement Among Adolescent Girls. (NIE-G-79-0159)

Future Directions

The problems discussed in this paper are very complex and will require a multitude of efforts if they are to be solved. Mathematics educators, parents, boards of education and the general public all have a responsibility in this problem-solving endeavor.

A major research thrust is needed to document what the actual problems are, why they exist, and the extent of their existence. Definitive reasons are needed as to why black students are failing to enroll in the mathematics courses so important for success in a mathematics related career. Are parents, counselors, teachers, and others really aware of the importance of mathematics to the student's future? What actually happens in the elementary school that contributes to a student's decision to take only the minimum number of mathematics courses needed for high school graduation? Are teachers prepared to teach to each student's strengths and show evidence of realizing that children process mathematics in many different ways? It is encouraging to see NIE begin to fund research in this area but a major commitment of resources is needed.

Research paradigms that include sociological factors and consider the total context in which learning occurs must be utilized. Cross-discipline teams of researchers, including minority researchers, must be formed. Further, a new look at the usefulness of typical, experimental-control designs using group data must be taken. Many of the questions posed here must be answered by clinical studies involving individual students through a case-study method. Longitudinal studies are also called for. New instruments must be designed; instruments sensitive to the idiosyncrasies of black students. Matthews (1981) points out, correctly so, that we have few, if any, test instruments that have been standardized on black samples. Validity questions must be raised about many instruments currently being used.

The results of our research must be disseminated effectively to parents, teachers, and others who influence the decision making process of black students. As we wait for the results of research currently in progress, mathematics educators must increase their efforts to educate parents and the general public of the mathematical plight of black students. Mathematical educators should take the lead in demanding that teachers teach mathematics to all their students and that educational practices which impact negatively on black students, such as inflexible "ability grouping" and the use of admittedly biased standardized IQ and achievement tests be discontinued. Finally, teacher training programs must prepare teachers to teach in multicultural settings. Teachers need to know mathematics and how to relate to black students. This nation can no longer afford insensitive teachers who blame the victim for his or her underachievement!!!

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HISPANIC STUDENTS AND MATHEMATICS:

RESEARCH FINDINGS AND RECOMMENDATIONS

by

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HISPANIC STUDENTS AND MATHEMATICS

RESEARCH FINDINGS AND RECOMMENDATIONS

The Problem

The problem generating this paper can be simply stated as follows: A substantial majority of Hispanic students enrolled in public schools across the country have not achieved adequately with regard to established criteria, nor comparably with their white cohorts in mathematics as determined by standardized test or other means used to measure success, i.e., course grades, enrollment in advanced mathematics courses, percentages of math majors, degrees earned in mathematics, or representation in math-related occupations. Since the status of underachievement in schools by Hispanics has been well documented and generally known by educators, verification of this unacceptable status warrants only minor treatment in this paper.

In order to show the magnitude of the condition facing educators and being endured by past and present generations of Hispanic youngsters, enrollment numbers and projections of Hispanic student enrollment are helpful. In 1979, the U.S. Census Bureau reported that there were 3.5 million Hispanics between the ages of 5 and 17 years. A study conducted by the National Center for Educational Statistics projected that the 1980 2.4 million limited-English-proficient (LEP) children ages 5 to 14 will increase to 2.8 million in 1990 and to 3.4 million by the year 2000. Spanish LEPs move from 1.8 million or 71 percent of all LEPs in 1976 to 2.6 million or 77 percent of all LEPs in the year 2000. Concurrently with this projected increase of Hispanic student enrollment in public schools, two counter-productive phenomena continue to persist. One event is that Hispanic students continue to leave school prior to satisfactorily completing all twelve grades. For example, in 1971 the U.S. Commission on Civil Rights reported that 40.7 percent of Mexican Americans residing in the five southwestern states did not complete high school (Commission Report II). In 1976 or five years later, 9.6 was the median years of school completed by Hispanics while 12 years was the median for the total population (U.S. Census, 1977). While Mexican Americans are only one sub-group of the Hispanic population, they represent 65 percent of the total Hispanic population or the largest sub-group (Census, 1979).

Paralleling the early-withdrawal-from-school phenomenon is the under-representation of Hispanics in math and science courses in junior and senior high schools. While no data were found specifically showing Hispanic percentages of enrollment in elected or advanced math and science courses, two studies lend some support for under-enrollment in such classes. Results of the 1977-78 National Assessment of Educational Progress (NAEP) reveal that the majority of 17-year olds took only two years of math courses and black students appeared to take about one year less of high school mathematics (Anick, Carpenter & Smith, 1981). Hispanic enrollment in math courses probably reflects more the black student pattern than the white student enrollment. In a study conducted in Tucson, Arizona with focus on attitudes toward science, MacCorquodale (Note 1) concluded that many Mexican American junior high students stop taking math and science courses after they had completed the minimum requirements.

With regard to mathematics achievement by Hispanics, the second mathematics assessment by NAEP graphically showed that in terms of student outcomes, serious inequities exist in the mathematics education of Hispanics and blacks. For 9-year old Hispanics, the NAEP indicated they were 9 percentage points behind the national average; 13-year old Hispanics were 13 percentages below, and 17-year old Hispanics were 12 percentage points behind the national average (Anick et al., 1981). Assessment by states of Hispanic math achievement may reveal a worse set of scores. For example, Texas has approximately 974,000 Mexican American students enrolled out of 3 million or 30 percent, and the 1980-81 results of a state mandated testing program called Texas Assessment of Basic Skills (TABS) show that the percentage of Hispanics achieving mastery in mathematics in grades nine and ten are 60 and 46 respectively. In comparison to their white Texas grade-level peers, Mexican American are 24 and 9 percentage points behind at each grade level assessed (TEA, 1981).

Before leaving the discussion of mathematics achievement by Hispanics, it is important to point out early that one trend appears to exist even though the above statistics do not reveal it: While Hispanics are achieving less in math in comparison to their white cohorts, they are scoring better than most other minority groups with the exception of Asian Americans. Also when comparing Hispanic math scores to other scores made by Hispanics on skills such as reading or content areas such as social studies, the math scores appear to be slightly better for Hispanics.

Finally, documentation of inadequate participation in mathematics by Hispanics can be illustrated by under-representation in math-related occupations. As with other minority group members, Hispanic professionals are concentrated in education, public affairs and the social sciences (National Science Foundation, 1980). More telling, of the 44,800 mathematicians reported in the 1970 labor force, only 637 or 1.3 percent were of Spanish surname origin (Vetter & Babco, 1975).

All of the above information goes toward characterizing the problem to be: Hispanics are not proportionately participating in mathematics education nor are those few who are partaking in mathematics doing so to a satisfactory level. Consequently, Hispanics are grossly under-represented in mathematics-related careers and as a result inequities in society continue to prevail.

Rooting the Problem

Since the problem in the foregoing discussion has been defined to be three-fold (disproportional participation in mathematics education, under-achievement in math, and under-representation in math-related careers), it seems wise to separate the discussion about possible causes to coincide with each of these three problem areas.

As has tried to be shown, Hispanic student underparticipation in math education is only one problem of the greater concern, i.e., early exit from schooling and general underachievement in most academic subjects. The question of why the holding power of schools with regard to Hispanics is weak

has been responded to in multiple ways. The U.S. Commission on Civil Rights, 1971-74 studies (probably the most comprehensive study conducted on any Hispanic group) on the Mexican American experience in schools in the southwest ties the weak school holding power to negative attitudes and treatment by school personnel toward Mexican American students. Briefly but specifically, the Commission's study carefully documents the following unjust practices: creating and maintaining inadequate and segregated school facilities, disproportionally high school suspensions, excess amount of corporal punishment, exclusion from extracurricular activities, unwarranted placement in EMR classes, high concentration of placement into non-academic tracks, over-reliance on grade retention, and until recently, the prohibition of the Spanish language on school grounds. Furthermore, the U.S. Commission and other researchers contribute the results of such unfavorable treatment to be motivated by prejudicial attitudes, stereotypic thoughts, and racial feelings harbored by school administrators and teachers toward Hispanic and other minority group members. Such cognitive and affective attributes do not solely reside with educators. Legislators also manifest such beliefs by drafting and maintaining state laws which skew school finance formulae against districts enrolling a majority of minority youngsters. Laymen elected or appointed to hold state or local school board positions also institute policies which adversely affect minority student populations, like adopting an English-as-a-second-language program only for LEPs rather than a bilingual education program.

Discontinuation of the above practices has occured only recently, unfortunately not sufficiently and primarily sporadically. While these unwanted policies, practices and treatment have been reduced, mainly due to litigation, favorable alternatives have been slow to emerge. More about constructive pedagogical approaches will be presented later in this paper.

Before discussing possible explanations for Hispanic under-achievement in math education, discussion will address the under-representation of Hispanics in mathematic-related careers. As was presented earlier, and will be elaborated on in the next section, the disproportionately large number of Hispanics not participating in math-related careers is primarily due to the inadequate accumulation of math education or an insufficient amount of math preparation. It is important to examine the factor of motivation when discussing persistence, especially when viewing the subject of mathematics. Contrary to the commonly held perception that minority students have lower occupational aspirations than their white peers, research by Anderson and Johnson (1971), Espinoza, Fernandez and Dornbusch (1977), and Juarez and Kuvlesky (1969) indicate the opposite. Furthermore, these same researchers found that parents of Hispanics view the importance of education for their children as much as Anglo parents. Not only do Hispanics consider school to be of high value, but they also link success in school with success in the world of work (Espinoza et al., 1977). When MacCorquodale (Note 1) asked junior high school students in Tucson how important science was in terms of the work they expected to do, Mexican American males perceived science as the most important. While no studies were found providing information about Hispanic students' attitudes toward mathematics, Anick et al. (1981), when analyzing NAEP data, found that black students like math, thought it was important, wanted to do well and would work hard to do so, in addition to wanting to take more mathematics and even entering a career using the

subject. While no direct relationship can be made between black attitudes toward math and Hispanic students, it is encouraging to note that Hispanic educational status and experience are most similar to the black education experience. Consequently, the small amount of studies available seem to indicate the problem locus of under-representation of Hispanics in math-related careers is not within the Hispanic personality.

Underinclusion of Hispanics in math-related careers can better be attributed to external variables like the lack of role models. Role models are important since their presence indicates to students the likelihood of their entering the profession. Moreover, role models have the opportunity to become mentors, identifying and encouraging students to move toward certain careers. As students enter college and come closer to entering the profession, role models can become sponsors, providing aspirants with counsel about how to acquire a position. While the function of counseling can and is assumed by role models, it is directly and primarily the task of school counselors. Yet, experience, literature on school counselors, and studies conducted on school counseling reveal that because of high student to counselor ratios and traditional stereotypic views of minorities, Hispanics have minimal contact with counselors, and what little time is spent with them is not devoted to educational and/or career planning.

Finally, a small number of Hispanics are found in math-related careers because of discriminatory hiring practices and because of minimal career information provided by schools. Students in rural schools and barrios of large urban cities are poorly informed about the various kinds of occupational roles and the educational requirements for most of these work stations.

Generally, discussion about the probable factors causing underachievement of Hispanics in mathematics is usually speculative in nature. Discussion of likely causes is not grounded in empirical findings simply because there has been very little research conducted to uncover the variables producing this inequitable learning conditon. However, while research is not available, the literature seems to be consistent in identifying the most likely factors and even consensus exists as to what intervention needs to be pursued if this underachievement is to be eliminated.

A review of the literature has identified five probable factors which could explain the underachievement status recorded by Hispanics in mathematics education. The five factors are language, cognitive learning styles, instructional methods, curricular material and teacher adequacy. While each of these factors will be discussed separately, in practice they are interrelated, and consequently it is difficult to determine which contributes and to what degree, to learning or lack of learning.

Language is a manifestation of culture; it both represents a group's society and creates present and future reality for the individual. With regard to learning, at minimum language is a mediator of concept development, and at most a determinant of concept formation. Concepts are categories of generalizable or related facts. Language is a coherent set of symbols used to label concepts and provide a quick reference and meaning to concepts.

Concept learning at some stage involves word association between a concept and the concept name (Underwood, 1966). Discussion of concept development is relevant because as is obvious, Hispanics have a culture and language (Spanish) which is different from mainstream America. Also remember that of the 3.5 million Hispanic students approximately 2 million are limited-English-proficient. Consequently, it is crucial that mathematic concepts be presented to LEPs in the language they are most proficient in. It is well known that there are few bilingual teachers able to communicate with and instruct children proficient in a language other than English.

The phenomenon of culture connects the language instruction variable with the curriculum variable. While most educators agree that mathematics is the subject most culture-free, it is not devoid of representing cultural biases. For example, Bradley (1981) asks to what extent is mathematics culturally biased since logic is a major strand in mathematics, and b) is not logic organized differently by cultural groups? Furthermore, according to Maria (1961:6), "mathematics is concerned with the study of relationships of ideas that man has abstracted from observation of the physical world and has generalized by rational reflection." Not all cultures interpret the physical world nor generalize about it in consistent ways. Differences between cutural groups has implications to the teaching of mathematics in two areas: curriculum materials and instruction. The way math concepts are presented in textbooks may be inconsistent with how immigrant Hispanic students have been introduced to certain concepts (Castellanos, 1981). This inconsistency may cause confusion in the student's mind and delay his/her understanding. Also, Tsang (1981) states that the curriculum should reinforce what has been learned previously and introduce new concepts based on previous knowledge. Immigrant students come from different cultures and are products of the educational system of their respective countries. The curriculum revision effort of the last two decades in the U.S. has led to significant differences between the mathematics curriculum of the U.S. and those of many other countries. Finally, with regard to the teaching of mathematics, Lovett (1981) while indicating that math is culture-free, states that the teaching of math takes place in a cultural context. Moreover, he continues that good math teachers have searched for the mathematics existing within the real world experiences of their students and have tried to incorporate such experiences in their teaching.

The amount of learning acquired by any student is most directly correlated to teacher quality. Castaneda (1981), along with others such as Gallegos (1981) and Serna (1981), argues that teachers' attitudes toward mathematics and the teaching of mathematics need to be changed to be more positive and constructive. She also states that teacher training should include mathematics content, information about the learning characteristics of young children, and methods of mathematics teaching. Such training would help teachers to combine what they know about math with their knowledge of pupils' learning traits, and to acquire skills that would facilitate selection of math content and instruction that honor the young child's natural modes of learning.

Possibly, the most potent factor related to achievement and consequently having the greatest promise for program intervention and remediation is the cognitive styles of Hispanics. While Castaneda only argues for teachers to

be prepared to match student learning characteristics with math concepts and methodology, Ramirez and Castaneda (1974) propose a specific cognitive style which may map with Hispanics and therefore have significant pedagogical implications. As a result of their research, Ramirez and Castaneda suggest that children from certain cultural groups may be more field dependent, i.e., view their environment as unified and having an inherent order. In contrast, field independent persons appear to view situations in distinct parts and apply structure even when no order is readily apparent. Under this dichotomy, Hispanics tend to be more field dependent. Lovett (1981), in his writing about the mathematic teacher's role in bilingual education, points out that there are three aspects of field dependency which have direct bearing on the teaching of mathematics. One, field dependent students may learn math more readily where the teacher provides a high degree of guidance, clearly states the outcomes, and structures the lesson. Two, presentation of materials or materials favoring social content seem to be more attractive to field dependent pupils. And three, working together and minimizing competition is a learning mode more conducive for field dependent youngsters. It appears that mathematics education has been organized to favor the field independent rather than the field dependent, i.e. open ended discovery rather than definite outcomes, individualization of instruction rather than group learning, and competition more so than cooperative activities.

Intervention Strategies: Uprooting the Problem

An effort to identify intervention programs attempting to remediate the banal learning of Hispanics in mathematics produced no results. While no reports were found in the literature, there have been and probably are a few experimental math programs with focus on Hispanics. Most of these few programmatic efforts have been directed at curriculum development and have been school district initiated. It is safe to state that no prominent math project has been designed and implemented with the primary purpose of testing teaching strategies applicable to Hispanics learning mathematics better. The search did, however, uncover a potpourri of studies and and a few proposals.

Trevino (Note 2), in her 1968 dissertation study which assessed the effectiveness of a bilingual program in the teaching of mathematics in the primary grades in one south Texas school district, found that in all cases the scores of Spanish-speaking children taught bilingually were higher than the scores of Spanish-speaking children taught exclusively in English. This finding is supported by two studies, one conducted by Coffland and Cuevas (1979) and the other by DeAvila (1979). It is also the most consistent finding, viz., bilingual students do better in math achievement than LEP and monolingual English-speaking students when taught in their mother-tongue and English. This finding even extends to the college-age pupil. Gerace and Mestre (Note 3) are examining college-age Spanish-speaking students majoring in technical subjects and report that their performance on mathematical skills is strongly related to language proficiency, more so than for monolinguals.

Research presently being funded by NIE provides educators and curriculum writers with insight about what researchers consider to be necessary knowledge in order to design instructional programs which will overcome the underachievement of Hispanics in mathematics. The team of Gerace and Mestre

(Note 3) is investigating the cognitive processes of ten ninth grade Hispanic bilingual students learning Algebra I. They anticipate that their findings will aid in the designing of math curricula for bilingual students and in helping bilingual students become aware of successful strategies and common avoidable errors. In other research, Orvik (Note 4) is directing an interdisciplinary research effort into the nature of mathematics teaching and learning across cultures leading to the generation of hypotheses about the cognitive characteristics of learners and tachers; the nature of mathematical tasks; and teacher/learner social interaction in cross-cultural situations. Still another researcher, Saxe (Note 4) is examining monolingual and bilingual Spanish/English and Chinese/English children ages 3-14 from middle and lower class families. Among many anticipated findings, he hopes to discover special competencies of inner city bilingual children and to illuminate early sources of differences in levels of preparedness of children to engage in math learning. There are other studies directed at non-Hispanic populations but which have bearing on aspects pertinent to Hispanics. Such research will not be discussed here because of the lack of space.

Besides the scant research mentioned, two proposed intervention programs were found. Hernandez (Note 5) expresses dissatisfaction with most research being conducted with regard to Hispanics and mathematics because such effort is aimed at identifying the primary cause of underachievement and diverts attention from designing programs to increase math achievement for Hispanics. Instead, she proposes a model for change based on positive directions to improve instruction. Specifically, she argues for Bloom's Learning for Mastery model as an approach that can be implemented now to ensure increased mathematics achievement. Possibly having the same motivation as Hernandez, Ortiz-Franco (Note 6) makes some suggestions for inclusion when developing an intervention program for precollege minority students who are interested in pursuing math/science careers. His suggestions are not based on research targeted at Hispanics, but based on intervention programs designed to increase the participation of white women in mathematics-related careers. In general, he discusses internal and external variables found to be important in intervention programs but comments correctly that specific implementation of such variables are dependent upon on-site circumstances, consequently offering none.

Consequently, a review of the scant research and literature reveals findings about how to improve the academic learning of Hispanics in mathematics are minimal and knowledge of remedial instructional programs suited to help Hispanics learn more mathematics is even less.

Research and Program Recommendations

It is recommended that a balanced and concurrent two-pronged approach be taken to bring about improved mathematics education to Hispanic students. One prong, the major of the two, be research and the second prong be experimental projects. As the research component builds a substantial volume of knowledge, it should reduce in importance and the second prong becomes the major effort, i.e., program implementation. The research focus should revolve around three dimensions: a) the Hispanic child's cognitive and affective style, b) methods of teaching mathematics to Hispanics and c)

scope, sequence and format of curriculum. Since all three of these dimensions of learning, methodology and curriculum are interrelated, they should be treated as such. However, joint investigation should not be at the exclusion of examining one dimension unto itself.

Within each of the three research foci, studies should explore the following particular aspects. Of course, the questions are not all inclusive nor prioritized. With regard to cognitive and affective learning styles of Hispanics, the following questions seem critical:

1. Is a bilingual's problem solving ability dependent upon first or second language proficiency?
2. What knowledge of mathematical concepts do Hispanic children have prior to entering school? What modes of motivation encourage Hispanics to pursue learning? Under what circumstances do Hispanics learn math concepts at home?
3. How do Hispanics approach learning?
4. How does a Hispanic child's difficulties in math depend on a particular concept rather than on a lack of general aptitude?
5. What are the psychological factors which influence Hispanic orientation to math? (This question would include the avoidance and persistance variables.)
6. What concepts, skills and predispositions are associated with succes and pleasure in math learning by Hispanics?

With regard to teaching methods, the following questions might be most fruitful:

1. How does the classroom environment influence Hispanic achievement?
2. What teacher behaviors promote positive interaction with Hispanic students?
3. What instruments and procedures can teachers use to identify cognitive learning styles of Hispanics?
4. How does the school climate and structure affect the Hispanic student's desire to participate and pursue a mathematics major?

Rather than research a series of questions when attending to curriculum, it is proposed that it would be more advantageous to undertake a developmental approach. The developmental thrust would be to generate curricula which incorporate the Hispanic's history, present environment and cultural values.

The second prong, project experimentation, could center around designing and implementing Bloom's model of mastery learning or incorporating Ramirez and Castaneda's constructs favoring field dependent learners. Such experimental projects would have to be well funded and participating staff as well as support staff would require extensive training. Of course, such experimental projects would necessitate a systematic evaluation plan, both formative and summative.

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ERIC/SMEAC Mathematics Education Fact Sheet Number 1 **1981**

Females and Mathematics

Attention in recent years has been directed toward the status of girls and women in relation to mathematics. How much mathematics to they take? How are they treated in mathematics courses? How do they perceive mathematics? Answers from research and other literature to these and other questions are summarized in this fact sheet. Additional information can be obtained from a variety of sources, many cited on a recent bibliography (Suydam and Kirschner, 1981). Two sources were of particular value in developing this summary. The first, prepared by Armstrong (1980), is an overview of the results of a two-year study conducted by the Education Commission of the States as part of the Women in Mathematics program. The second is an ERIC/SMEAC publication, *Perspectives on Women and Mathematics*, edited by Jacobs (1978), ED 166 051.

Female Participation in High School Mathematics

Only a small percentage of women has pursued a mathematics-related vocation, such as engineering. Researchers have ascertained that most females are not prepared to enter such vocations because they lack the necessary mathematics background. Furthermore, it appears that in many instances girls have not chosen to enroll in advanced mathematics courses in high school, and have received insufficient encouragement or support to take such courses. The College Entrance Examination reported in 1978 that 63 percent of college-bound males had taken four or more years of high school mathematics, but only 43 percent of college-bound females had done so.

Results from both the Second Mathematics Assessment of the National Assessment of Educational Progress (Carpenter et al., 1981) and the Women in Mathematics survey (Armstrong, 1980) showed that male and female participation was similar for basic mathematics courses: general mathematics, algebra I, and geometry. The pattern changed for more advanced courses, however. In the NAEP assessment, statistically significant differences favored males in enrollment for trigonometry and for pre-calculus/calculus, while the Women in Mathematics survey reported significant differences favoring males in enrollment for algebra II and for probability/statistics. Nonsignificant differences in enrollment for trigonometry and for pre-calculus/calculus also favored males.

Another study similarly indicated that, beginning at about the level of algebra II and continuing beyond high school, girls increasingly decided not to study mathematics (Fennema, 1977).

Peer Influences on Female Participation

Females who chose to pursue advanced mathematics courses were found to receive less attention from their male peers (Luchins, 1976) and to feel uncomfortable with them in social situations (Fox, 1976). Since boys in grades 6 through 12 stereotype mathematics as a male discipline at a significantly higher level than do girls, it has been suggested that boys communicate this attitude to girls in various ways (Fennema, 1978). The Women in Mathematics survey found that in the twelfth grade peer influence had a significant correlation, albeit low, with participation in mathematics (Armstrong, 1980).

Differential Treatment by Teachers

Several researchers have found that teachers treat boys and girls differently in mathematics classes (Bean, 1976; Parsons et al., 1979). High-achieving boys received significantly more attention in high school mathematics classes than did other boys and girls, including high-achieving girls (Good et al., 1973). Furthermore, students tend to be influenced by what they believe the teacher thinks of them and their ability in mathematics Fennema and Sherman, 1976). There is also evidence that teachers expect sex-related differences in achievement (Fennema, 1978).

A study on teacher-student interaction in high school geometry classes reported differential treatment on such factors as offered responses, cognitive level of questions, sustenance and persistence, praise and criticism, individual help, and even conversation and joking (Becker, 1981). In general, the interaction patterns reinforced the sex-typing of mathematics as a male domain. Becker hypothesized a three-step pattern:

(1) Teachers expect differences between male and female students.

(2) Teachers treat students differently on the basis of sex according to their differential expectations.

(3) Students respond differently in class consistent with teachers' and society's sex-role expectations.

Stereotyping Mathematics as a Male Domain

Historically, mathematics has been regarded as a masculine discipline. The results of the Women in Mathematics survey clearly indicated that mathematics is both a female and male domain. Thirteen-year-old girls were found to be better at spatial visualization and computation than were boys, while their problems, solving skills were nearly equal. By grade 12, girls' abilities in spatial visualization and computation were comparable to boys', but boys excelled in problem solving (Armstrong, 1980). These findings seem to support Fennema's (1978) conclusion that no inherent factors exist which keep girls from learning mathematics at the same level as boys.

The Women in Mathematics survey found that females who regard mathematics as a subject for both males and females tended to take more mathematics (Armstrong, 1980). In another study females in grades 6 through 12 denied the belief that mathematics is strictly for males (Fennema and Sherman, 1977).

Female Attitudes Towards Mathematics

In studies conducted 20 or more years ago, boys at the elementary school level seemed to prefer mathematics slightly more than did girls; that is, boys' attitudes were more favorable than girls' attitudes. In more recent studies, however, no overall sex differences in preference or attitudes have been observed at the elementary school level (e.g., Ernest et al., 1975).

On subscale measures of attitudes, however, differences are found, particularly at the secondary school level. Female students, to a lesser degree than males, viewed mathematics as personally useful (Fennema and Sherman, 1977). Research has also indicated that girls who regard mathematics as useful in their future are more likely to continue taking mathematics. For example, the Women in Mathematics survey found that twelfth-grade students who plan to continue their education beyond high school were more likely to take more advanced mathematics courses than were students with lower academic aspirations (Armstrong, 1980).

Anxiety and Confidence

As Armstrong (1980) stated, a student's attitude about mathematics reflects his or her enjoyment, confidence, and anxiety concerning the subject.

Low achievement in mathematics has been associated with high anxiety; furthermore, low anxiety has been correlated with a high degree of confidence. Fennema and Sherman (1977) found that at each grade level 6 through 12, boys were significantly more confident in their mathematical ability than were girls, even though there were no significant sex differences in achievement.

Parental Influences on Female Attitude and Achievement in Mathematics

Parents who use, enjoy, and have confidence in their ability to do mathematics have been shown to have a positive effect on students' attitudes toward the subject (Fox, 1976). Fox also found that the expectations of fathers influenced the mathematics achievement of girls. Several studies ascertained that parents, unfortunately, seem to have lower expectations for their daughters than for their sons (e.g., Casserly, 1975; Levine, 1976).

Intervention Strategies

Increasing female participation is viewed as a key strategy for combating the factors that cause girls to avoid mathematics. In addition, it is believed that sufficient female participation in high school mathematics courses will permit a woman to pursue a scientific or technical career on the basis of equal opportunity.

From the Women in Mathematics survey data, Armstrong (1980) identified three important variables that have an effect on mathematics participation:

- positive attitude toward mathematics
- perceived need for and usefulness of mathematics
- positive influences of significant other people (parents, teachers, counselors)

She suggested that intervention programs designed to increase participation should focus on ". . . instilling and sustaining the positive influences of these factors . . ." (p. ix).

Furthermore, 14 action steps discussed by Burton (1978) have been helpful in increasing participation. The recommendations are practical and relatively easy to implement in the classroom. For example, she suggested inviting guest speakers to mathematics classes and encouraging female students to encourage other female students.

Fennema et al. (1981) concluded that intervention programs aimed at resolving the inequities in the study of high school mathematics can be effective. For instance, the amount of mathematics female students planned to take was significantly increased in schools where the intervention program "Multiplying Options and Subtracting Bias" was in effect. Additional intervention programs are described in Jacobs (1978) (e.g., by Afflack, Liff, and Tobias).

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ERIC/SMEAC Mathematics Education Fact Sheet Number 1 **1982**

Helping Low-Achieving Students in Mathematics

All of us know that some children achieve less than they might, or less well than their peers, for a variety of reasons. We may give such children different labels if their lower achievement seems to be an outcome of such a factor as intelligence or a handicap. The nature of the cause *is* important for a teacher to determine, for it may indicate that a specific instructional method will or won't work. So we need to begin with the student, assessing his or her learning style, interests, and talents, and attempting to pinpoint the mathematical ideas with which difficulty exists. Diagnosis is imperative!

The next step is to search for instructional materials and strategies which appear to be appropriate to help the student at his or her particular stage of development. Unfortunately, few techniques will work with every child, in every instance. Fortunately, there are some guidelines, supported by both practice and research, that *do* seem applicable for remediation with many students:

1. Keep careful records and use diagnostic testing before and after teaching so that you can . . .
2. . . . analyze a child's difficulty specifically.
3. "Back-track" as far as necessary to redevelop ideas and skills . . .
4. . . . in a new way. Remediation is often simply a rehashing, with ideas presented in the same way the second or third time as the first time. Instead, aim to capture the child's interest, capitalize on what the child does best, build assignments which lead to success instead of repeated failures, and enhance the child's self-concept.
5. Spell out realistic, short-term goals and clear objectives, and . . .
6. . . . involve the child in planning those goals and ensuing instruction.
7. Make the content applicable to the child's needs, stressing the practical utility and relevance of mathematics.
8. Slow the pace of instruction, taking developmental needs into consideration, and structure instruction in small, sequential steps.
9. Employ diverse types of experiences and varied approaches, encouraging active involvement with a variety of manipulative materials at all ages.
10. Plan carefully to distribute practice and provide frequent reviews.
11. Give a maximum of explanation and guidance during lessons.
12. Give feedback and reinforcement as soon as possible.
13. Provide on-going evaluation: growth should be apparent to the child.
14. Use calculators: don't let computation be the stumbling block that keeps a child from going on to learn other mathematics.
15. Expect achievement from *every* child, and show confidence in his or her ability to learn.

Research has indicated that one of the major variables in determining achievement is time, and in particular how much time a student spends "on task" — actually doing mathematics. Consequently, if you want to enhance achievement, attend to this by assuring that children are kept involved in the lesson, both individually and in group work.

How a child feels about his or her ability to do mathematics appears to be another major variable, and remediation should consider the affective as well as the cognitive. Sobel (1959) commented on junior high school students, but his comment seems applicable to students at other levels as well:

> . . . the slow learner in the junior high school has the same characteristics as other pupils of the same age, the same basic needs and interests. However, more than the average child, he needs to be given the chance to experience success and approval: more than the others he needs to feel that he is a member of a group with a contribution to make; he needs status; his confidence must be re-established, his interest stimulated, his attitude toward mathematics made favorable, his ego flattered. (p. 348)

In many of the references that follow, both the cognitive and the affective status of the child are mentioned. The references were selected because they focus on children who are having difficulty learning mathematics, whether because of intelligence, motivation, or disability. They cover a range of levels from elementary through secondary school. In addition, there are many other references (in ERIC and elsewhere) that present activities for higher achievers that would also be appropriate (possibly with some slight adjustment) for these students.

Finally, as you plan instruction, consider this suggestion from *An Agenda for Action: Recommendations for School Mathematics of the 1980s:*

> In many current programs, the student who does poorly in the algorithmic skills finds progress in all aspects of mathematical development halted, since remediation is designed to concentrate solely on this deficiency. Remedial programs should identify other areas of mathematical ability — for example, spatially related skills — and concentrate attention also on the students' strengths, not solely on their deficits. (NCTM, 1980, p. 18)

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Sobel, Max. "Providing for the Slow Learner in the Junior High School." *Mathematics Teacher* 52: 347-358; May 1959. Suggestions are given for teaching junior high school students, with an emphasis on motivation.

Suydam, Marilyn N. (ed.). *Developing Computational Skills.* 1978 Yearbook. Reston, VA: NCTM, 1978. Included are chapters by Backman on analyzing errors, Inskeep on diagnosing difficulties, and Moyer and Moyer on teaching suggestions for learning disabled children.

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Resource Aid of Selected Materials for Remediation of Learning Disorders. Boston: Boston University, 1971. ERIC: ED 052 580. Forming diagnostic profiles and classifying materials to match areas of strength and weakness in learning are discussed.

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SELECTED PROGRAMS

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PROGRAM

EQUALS

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PROGRAM DESCRIPTION

EQUALS encompasses programs for teachers, counselors, administrators, parents, and scientific and technical workers. It promotes participation of students and adults in mathematics courses and encourages their interest and involvement in math-based fields of study and work.

Since 1977, 1,500 California educators and 2,000 educators from 25 other states have participated in EQUALS programs. Thirty staff developers from 20 states have taken EQUALS training and implemented the programs in their states.

Evaluation data indicate our programs promote increased student enrollment in advanced mathematics classes; improved student attitudes toward the study of mathematics; increased student interest in math-related occupations; and enhanced professional growth of teachers.

Programs Include:

EQUALS

- Offers staff development for teachers, counselors, and administrators serving Kindergarten through College;
- Provide materials to attract and retain students in mathematics classes;

EQUALS IN TECHNOLOGY

- Develops a sense of mastery over the powerful new technology available for educational use today;
- Assists participants in evaluating the appropriate use of computers and calculators

- Models a variety of instructional techniques to improve student attitudes toward mathematics;
- Increases awareness of career options and interests in math-related occupations
- Enables educators to integrate computers and calculators into the curriculum.

SPACES

(Solving Problems of Access to Careers in Engineering and Science)

- Provides classroom activities relating mathematics to careers;
- Improves students' problem-solving skills, including spatial reasoning;
- Give students career experiences through simulations.

THE MATH/SCIENCE NETWORK

- Promotes interaction and cooperation between the educational community and business and industry;
- Encourages women to enter non-traditional math- and science-based occupations.
- Links professional trainers and model programs with appropriate audiences.

FAMILY MATH

- Teaches parents how to help their children with math at home;
- Informs parents of the role mathematics will play in their children's studies and choices of careers;
- Creates family enjoyment of mathematics.

MATERIALS AVAILABLE

Use EQUALS to Promote the Participation of Women in Mathematics

Handbook describes the EQUALS teacher education program and provides methods and materials for educators at the elementary and secondary level, as well as in pre-service courses. Annotated bibliographies on problem solving in mathematics and sex-fair counseling and instruction. 134 pp. $7.50.

Math for Girls and Other Problem Solvers

Curriculum for Lawrence Hall of Science course, "Math for Girls," with student activities and game sheets. Appropriate for elementary and secondary students. 108 pp. $7.50

Women Moving Up

Designed for women getting started and moving up in scientific and

SPACES

Thirty-two classroom activities to help students' problem-solving abilities while increasing their knowledge about scientific and technical fields of study and work. Suitable for grades 4-10. 141 pp. $10.00

I'm Madly in Love with Electricity and Other Comments About Their Work by Women in Science and Engineering

A career booklet for young people curious about the sciences. Includes comments from 70 women scientists, engineers, and mathematicians about the work they do and the opportunities in their fields. 73 pp. $2.00

technical fields. Profiles 25 employers of scientists, engineers, and technicians, and women working in these fields. 133 pp. $5.00

To order publications, please enclose a check payable to The Regents, University of California, and mail to EQUALS. Shipping and handling: $1.00 per book. California residents add 6% sales tax (BART area, 6½).

STATE

WASHINGTON

INSTITUTION

SOUTH SEATTLE COMMUNITY COLLEGE
Public, two-year
Coeducational
Enrollment: 7,800

PROGRAM

MATH ANXIETY PROGRAMS
South Seattle Community College
6000 Sixteenth Ave. SW
Seattle, Washington 98106

(206) 754-5300

Contact Person(s): Jerine Ridgway, Mathematics

PROGRAM DESCRIPTION

There are three concurrent activities going on under the general rubric of Math Anxiety programs at South Seattle Community College, all under the direction of Jerine Ridgway who is trained both in mathematics and social work. One is a Math Anxiety Lab which is offered once per quarter (see Sample); the second, a Math Anxiety Clinic Workshop also offered every quarter, and the third, a Math Anxiety Clinic offered during the summer only as a part of an Algebra Review Course.

The program was initiated by Ms. Ridgway in 1975 by way of a supplementary "math anxiety lab" attached to a Saturday Algebra Review class. Since then, the material has also been used in many 3-4 hour workshops, as part of the Math Lab, and as informational material for lectures to schools and community groups. Current enrollment in the Math Lab course plus the numbers that entered the Algebra Review and Math Anxiety Clinic course in the summer of 1979 indicate a continuing need for the variety of courses as now structured.

Funding comes from regular state moneys used for all courses. Workshops, however, are funded by some sponsoring group such as Women's Programs, Home and Community Education, Senior Citizens, etc.

The Math Lab and math classes are designed to take place over the full quarter and each offers one credit of non-transferable identification. Some workshops are free to students; others have to charge all community participants. What is particularly gratifying are the numbers of women who are beginning to select these courses and the numbers of mathematics instructors in the Washington State area who are beginning to work up math anxiety programs. A list of math instructors whose programs may not yet be ready to be described but who are developing similar activities to those at South Seattle follows: Dorothy Crepin, Lower Columbia Community College; Ramy Hough, North Seattle Community College; Janet Ray, Seattle Central Community College; Mary Ann Doe, Shoreline Community College; Susan Bates, Malaspina College, British Columbia (all mathematics instructors); Joan Poliak, Bellevue Community College, and Pat Sherman, Tacoma Community College (counselors); Margaret Berry, Port Townsend School District (school superintendent).

MATERIALS AVAILABLE

Case studies to raise consciousness of teachers. 50¢

STATE

WASHINGTON

INSTITUTION

MATH COUNSELING INSTITUTE
Private service

PROGRAM

MATH COUNSELING INSTITUTE
4518 Corliss Avenue North
Seattle, Washington 98103

(206) 632-8639 or 323-4106

Contact Person(s): Peter Blum

PROGRAM DESCRIPTION

The Math Counseling Institute is a private teaching and counseling service offering small classes, eight students at a maximum, and one-to-one tutoring and counseling for adults and children.

One course, called Overcoming Math Aversion--for Adults deals with the "feeling blocks" which keep people from learning math. Emphasis is on setting realistic goals in a supportive atmosphere with work on some specific math problems. Each class is 1½ hours in length.

Overcoming Math Aversion--for Children uses the same basic format as the adult class but is geared to the needs of children. Each class is 1 hour long.

Individual sessions for both adults and children are also offered.

In addition, the MCI staff makes presentations to educational and community organizations.

The director, Peter Blum, has a Ph.D. in mathematics from the University of California at Berkeley and has taught at Columbia University in New York. He brings to his work three years of clinical training in transactional analysis and has been working in the area of math anxiety for four years. In addition to transactional analysis techniques, he employs gestalt techniques, and neuro-linguistic programming.

MATERIALS AVAILABLE

Descriptive brochure, on request.

A self-help book by Peter Blum is in progress. Write if you wish to be notified when it becomes available.

STATE

WASHINGTON

INSTITUTION

SHORELINE COMMUNITY COLLEGE
State, two-year
Coeducational
Enrollment: 8,242

PROGRAM

"I CAN'T DO MATH"
Shoreline Community College
16101 Greenwood Avenue North
Seattle, Washington 98133

(206) 546-4606/4776

Contact Person(s): Dianne Dailey
Mary Bennett

PROGRAM DESCRIPTION

Shoreline is a public community college that operates under the regulations of the Washington State Board for Community College Education.

The Women's Program, a department under Student Services, and the Women Student's Network first offered an "I Can't Do Math" workshop Winter Quarter 1979. Thirty-five people attended this half-day session, taught by Jerine Ridgway, guest instructor from Seattle Central Community College. Two of Shoreline's business math instructors, Maryann Doe and Michael Warlum attended this session and with encouragement from Ms. Ridgway, developed a model for a second math workshop. Twenty people attended the second session held Spring Quarter 1979.

Shoreline's Women's Program has continued to offer the math anxiety workshop once a quarter and has modified the program to be presented in 1½ hour sessions once a week for four weeks. Fifteen to twenty students attended each workshop. Evaluation forms and informal follow-up indicate that participants feel they have gained valuable information and anxiety reduction techniques. Most students successfully continue on in math courses.

Women's Programs is now working on designing a math discovery class to include both Math Anxiety Reduction and basic math skills.

MATERIALS AVAILABLE

Business Math - A Positive Approach by Mike Warlum and Maryann Doe, Goodyear Publishing Company. Write for details.

STATE

WASHINGTON

INSTITUTION

THE EVERGREEN STATE COLLEGE
State, four-year
Coeducational
Enrollment: 25,000

PROGRAM

OVERCOMING MATH ANXIETIES
The Evergreen State College
Olympia, Washington 98505

(206) 866-6000

Contact Person(s): Kaye V. Ladd, Chemistry
Hazel Jo Reed, Mathematics

PROGRAM DESCRIPTION

Work in Overcoming Math Anxieties is incorporated into some first and second year programs involving science. In addition the college has offered special 4-8 quarter-hour courses specifically designed to overcome math anxieties. These courses will be offered in the future on an as-needed basis. For information contact Stella Jacobs, Learning Resources Center, 866-6420. The following is a description of an 8 quarter-hour course as most recently taught:

Overcoming Math Anxieties is a half-time program which meets twice a week. The program is designed to work in three areas: math anxieties, thinking processes and college algebra.

Problem-solving covers the bulk of topics in college algebra: percents, fractions, exponents (power and roots), monomials and polynomials, solution of monomials of one variable, solutions of monomials of two variables, and applications of algebra to "word problems."

In addition, students learn to use algebraic calculators.

Students were invited to supplement their assigned work with either the self-paced text Practical Algebra by Peter Selby or the self-paced lessons on PLATO (a form of computer-assigned instruction) and were given two tests on the material.

Thinking processes were developed through reading and discussion of books (see below) and solving thought problems such as riddles.

Math anxieties were dealt with through direct discussion, book discussions, specific exercises designed to overcoming anxieties, and a journal.

The books assigned were: P. Anderson, The Trouble Twisters; E. Abbot, Flatland; T. Barville, How to Listen--How To Be Heard; C. Doyle, Adventures of Sherlock Holmes and Hound of the Baskervilles; D. Huff, How to Lie with Statistics and How to Take A Chance; E. Newman, Strictly Speaking; S. Tobias, Overcoming Math Anxiety; W. Sawyer, Mathematicians' Delight.

MATERIALS AVAILABLE

A five-page discussion of "Overcoming Math Anxieties" with instructions for teachers who wish to work with anxiety reduction, written by Kaye V. Ladd, available for $1.00 to cover xeroxing, mailing and handling.

Description of course as taught Fall 1980 available upon request. Contact Kaye V. Ladd.

STATE

WASHINGTON

INSTITUTION

UNIVERSITY OF WASHINGTON
State, four-year
Coeducational
Enrollment: 36,000

PROGRAM

IMPROVING TEACHERS' AND COUNSELORS' MATHEMATICS ABILITIES AND ATTITUDES
Mathematics Education
115 Miller Hall DQ 12
University of Washington
Seattle, Washington 98195

(206) 543-1847

Contact Person(s): Mildred Kersh, Mathematics Education
Nancy Cook, Psychology

PROGRAM DESCRIPTION

The program at the University of Washington has been inspired by a desire on the part of the co-directors to break the cycle of mathematics avoidance in females. Noting that many elementary education majors are females who avoided mathematics in high school and in college, they decided to focus on this population and to try to reawaken an interest in mathematics by building confidence and teaching mathematics creatively. The expectation is that participants' attitudes will change and that they, in turn, will be able to teach mathematics more creatively in the schools.

The project directors, currently supported by federal funds, (Fund for the Improvement of Postsecondary Education) manage three two-quarter courses each year. Each two-quarter course sequence has been between 20 and 30 students. Teachers and counselors (i.e. participants) receive three units per quarter and are charged $30.00 for both quarters.

No formal follow-up studies are planned. However, the former students from the grant program continue to meet with the staff on a monthly basis.

Mildred Kersh, Nancy Cook and their associates in the program have chosen to focus their courses, in part, on the development of and improvement of participants' spatial visualization abilities. This process involves: 1) a precise delineation of types of spatial abilities; 2) development of materials to stimulate and practice these abilities; 3) demonstration of use of these materials; 4) suggestions as to how to integrate them into the regular elementary or intermediate level math classroom.

The choice of spatial visualization as a skill-focus is derived from the allegation of lesser performance on spatial visualization tests of females than males and the putative connection between mathematical ability (or mathematical confidence) and spatial visualization.

In the course of the FIPSE grant period, a considerable array of spatial visualization materials were developed.

MATERIALS AVAILABLE

Spatial visualization kits, instructor manuals. Write for details.

STATE

CALIFORNIA

INSTITUTION

LEARNING INSTITUTE
1983 by Pitman Learning Inc.
The Learning Institute is a division of Pitman Learning Inc.
19 Davis Drive
Belmont, California 94002

PROGRAM

MEETING THE MATH CHALLENGE FOR THE 80's

PROGRAM DESCRIPTION

Enjoyable, Understandable, Practical. These words only begin to describe a Marilyn Burns math program. In five days of intense and exhilarating activity you will learn just how much fun a math program can be while you explore one of the most crucial aspects of your job--how to ensure that your students master computational skills and learn how to use these skills to solve problems. In short, how to make math make sense to your kids.

In five exciting, idea-filled days, you will learn how to:

. Provide activities that will help your kids enjoy math while they build skill and competence;
. Use straightforward, easily understood techniques to find out where a child's difficulty lies and what to do about it;
. Organize your classroom for maximum math learning;
. Meet the widely differing needs and abilities in your classroom without spreading yourself so thin that no one is helped;
. Break through to the kids who have given up and believe that math will forever remain a mystery;
. Ensure that your students master computation skills and learn how and where to use those skills to solve problems;
. Structure your math time to combat the "what do I do when I'm done?" syndrome;
. Help students help each other to solve math problems;
. Challenge the gifted math student;

- Organize and manage math supplies and materials more easily and efficiently;
- Broaden your math program to include more than basic arithmetic;
- Use a few simple principles to bring new clarity, effectiveness and power to your entire math program.....plus a host of innovative, classroom-tested strategies that you can use to tackle your toughest math challenges.

STATE

CALIFORNIA

INSTITUTION

CENTER FOR INNOVATION IN EDUCATION, Inc.
19225 Vineyard Lane
Saratoga, California 95070

PROGRAM

MATHEMATICS THEIR WAY and MATHEMATICS...A WAY OF THINKING Workshops

PROGRAM DESCRIPTION

The program is designed for classroom teachers, E.H. and L.D. teachers, para-professional aides, administrators, and parents who are searching for an alternative to the textbook for teaching mathematics with an emphasis on problem solving.

Each summer course is run as a model classroom with teachers actively involved in the learning process. The workshop focuses on manipulative materials and activity-centered learning. Its aid is to encourage thinking, understanding and creativity as well as mastery of basic skills.

Programs Include:

Mathematics Their Way
Workshop for
K-2 Teachers

Concepts dealt with:

Free exploration, pattern, sorting and classifying, comparing, counting, number (beginning addition and subtraction), measurement, graphing and problem solving.

Materials used:

Assorted "junk," recyclables, teacher-made cards and materials, unifix cubes, pattern blocks, wooden cubes, tiles, rice, milk carton scales, and geoboards.

Mathematics A Way of Thinking
Workshop for
3-6 Teachers

Concepts dealt with:

Establishing a learning environment for **all** students in class. Special emphasis on overcoming the difficulties of students who have trouble with math.

Materials used:

A variety of readily available, inexpensive materials which enable teachers to implement activity-centered learning on a very low budget.

Curriculum Resource Materials:

Mathematics THEIR Way

M. Baratta-Lorton

- Complete manipulative K-2 math curriculum
- 200-plus classroom-developed and proven experiences
- 600 photos of children involved in activities
- Sample teaching strategies, with suggested dialog
- Activities using familiar materials
- Easy-to-manage assessment system
- Typical teachers' questions answered
- Blackline masters for student worksheets

Mathematics...A Way of Thinking

R. Baratta-Lorton

- Detailed guide for an intermediate activity-centered math curriculum
- Successful learning experiences for students having trouble with math. Challenging math experiences for all students
- For whole-class teaching with students of widely varying math abilities
- Introduces concepts beyond arithmetic: problem solving, geometry, statistics, probability, and more
- Suggested dialog and step-by-step illustrations
- A complete curriculum, or activities to supplement any basal series
- No student reading; no student text

MATERIALS AVAILABLE

Mathematics Their Way	04321	$5.80
Replacement pad of Blackline Masters	04321	$5.80
Mathematics...A Way of Thinking (softbound)	04322	$15.00

To order publications, please enclose a check payable to:

Addison-Wesley Publishing Company
2725 Sand Hill Road, Room A204
Menlo Park, California 94025

STATE

VIRGINIA

INSTITUTION

PROGRAM

MULTIPLYING OPTIONS AND SUBTRACTING BIAS

Author: Elizabeth Fennema

PROGRAM DESCRIPTION

A videotape and workshop intervention program designed to eliminate sexism from mathematics education. Four 30-minute, full-color videotapes,

narrated by Marlo Thomas, each for a specific junior/senior high school audience, uses a variety of formats--candid interviews, dramatic vignettes, and expert testimony--to address the problem of mathematics avoidance and suggest some possible solutions. A 192-page facilitator's guide provides an overview of the workshops, detailed instructions on how the facilitator can prepare for conducting the workshops, and four separate step-by-step sets of workshop instructions for each of the target audiences. 1981.

MATERIALS AVAILABLE

Each Videotape & Guide.....$125.00
Students.....#303 Parents.....#304
Teachers.....#305 Guidance Counselors.....#306
All Four Videotapes and A Guide.....#307 $375.00
Specify tape size when ordering
3/4" Videotape, BETA ½" Open Reel & VHS

To order materials, please enclose a check payable to:

NCTNN Educational Materials
1906 Association Drive
Reston, Virginia 22091

STATE

CALIFORNIA

INSTITUTION

PROJECT SEED, INC.
Non-Profit Corporation

PROGRAM

PROJECT SEED
2336-A McKinley Ave.
Berkeley, California 94703

(415) 644-3422

Contact Person(s): William F. Jontz, Director

PROGRAM DESCRIPTION

Project SEED is a nationwide program in which professional mathematicians and scientists from major universities and research corporations teach abstract, conceptually oriented mathematics to full-sized classs of educationally disadvantaged elementary school children on a daily basis as a supplement to their regular arithmetic program. The mathematics is presented through the use of a Socratic group discovery format in which children discover mathematical concepts by answering a sequence of questions posed by the SEED instructor. The mathematical topics are chosen

from high school and college algebra to reinforce and improve the students' computational skills and to help equip them for success in college-preparatory mathematics courses at the secondary level. Project SEED teaches entire regular elementary school classes rather than specially selected groups of students.

Project SEED's long-range goal is to increase significantly the number of minority and educationally disadvantaged youth majoring in, and attaining careers in, mathematics and the mathematics-related fields. Project SEED attains this objective by raising the achievement level and consequently the self-concept of disadvantaged children by providing them with success in a high-status, abstract subject. It is important that the subject not be associated with past failure, as language arts and the more familiar arithmetic tend to be. The direct remediation which characterizes most compensatory education programs usually fails because it tends to derogate the child by concentrating on the areas in which he or she has already failed. Project SEED achieves success because remedial arithmetic is embedded in the SEED curriculum (high school and college algebra) which is new, high-status and free of past failure connotations.

The regular classroom teacher is always present when the SEED mathematician is working with his or her class. Consequently, Project SEED provides an ideal, ongoing, daily inservice training program for the teachers in whose classes we are working. Regular teachers learn mathematics and effective teaching methodology. They also acquire new expectations for disadvantaged children. This teacher training occurs far more readily in Project SEED than it does from traditional inservice classes because the SEED specialist demonstrates the new techniques and attitudes daily <u>in the regular teacher's own classroom.</u> A major evaluation of the California SEED project in more than 200 classes conducted by Cal Tech revealed a very positive attitude toward SEED on the part of the teachers in whose classes the program was operating.

RESOURCES

BOOKS

Armstrong, Jane M. **Achievement and Participation of Women in Mathematics: An Overview.** Denver, CO. Education Commission of the States, 1980.

Brush, Lorelei R. **Encouraging Girls in Mathematics: The Problem and the Solution.** Cambridge, MA. Abt Books, 1980.

Chinn, Phyllis. **Women in Science and Mathematics Bibliography.** Arcata, CA. Humboldt State University, 1980.

Fox, Lynn H. **Minorities and Mathematics.** New York, NY. Ford Foundation, Office of Reports, 1981.

Fox, Lynn H., Brody, L., and Tobin, D., eds. **Women and the Mathematical Mystique.** Baltimore, MD. The Johns Hopkins University Press, 1980.

Hall, P. Q. **Problems and Solutions in Education, Employment and Personal Choices of Minority Women in Science.** Washington, D.C. American Association for the Advancement of Science, 1981.

Humpreys, S., ed. **Women and Minorities in Science: Strategies for Increasing Participation.** Boulder, CO. Westview Press, 1982.

Lovett, James C. and Snyder, Ted. **Resources for Teaching Mathematics in Bilingual Classroom.** Ohio State University, 1979.

Skolnick, J., Langbort, C., and Day, L. **How to Encourage Girls in Math and Science.** Englewood Cliffs, NJ: Prentice-Hall, Inc., 1982.

Tobias, Sheila. **Paths to Programs for Intervention** ($16); **Resource Catalogue for Practitioners** ($13); **Resource Manual for Counselors/Math Instructors** ($12); **Self Help Kit for Students** ($12). (Order from: Institute for the Study of Anxiety in Learning, Washington School of Psychiatry, 1610 New Hampshire Avenue, NW, Washington, D.C. 20009. Telephone (202) 667-6380.)

ARTICLES

Armstrong, James M. **Achievement and Participation of Women in Mathematics Results of Two National Surveys.** Journal for Research in Mathematics Education, v12 n5 p356-72. November 1981.

Results of two national surveys indicate that sex differences in achievement on mathematical applications persist even when different levels of mathematics participation are controlled.

Becker, Joanne Rossi and Jacobs, Judith E. **Sex: Is It an Issue in Mathematics?** Educational Horizons, v61 p60-67. Winter 1983.

Becker and Jacobs point out that differences in mathematics achievement by males and females are influenced by differential societal expectations. They cite some intervention programs that have been successful in curbing the sex-typing of mathematics as male.

Benbow, Camilla Persson and Stanely, Julian C. **Sex Differences in Mathematical Ability: Fact or Artifact?** Science, v210 n4475 p1262-64. December 1980.

Substantial sex differences in mathematical reasoning ability in favor of males was apparently found by this study. The data was collected in an investigation of intellectually gifted junior high school pupils who took the Scholastic Aptitude Test.

Burton, Grace M. **Perception of the Horizontal and Vertical by College Students.** Journal for Research in Mathematics Education, v11 n5 p385-88. November 1980.

College students are assessed for their abilities at horizontal and vertical spatial tasks. Differences between males and females and between vertical and horizontal tests are covered in this study.

de Wolf, Virginia A. **High School Mathematics Preparation and Sex Differences in Quantitative Abilities.** Psychology of Women Quarterly, v5 n4 p555-67. Summer 1981.

On six mathematical subtests studied, males scored higher plus took significantly more algebra, geometry, advanced mathematics, and physics coursework. Females earned higher overall mathematics grades. After statistically controlling for the amount of coursework taken, sex differences disappeared on two quantitative tests and on spatial ability.

Fennema, Elizabeth; Carpenter, Thomas P. **Sex-Related Differences in Mathematics: Results from National Assessment.** Mathematics Teacher, v74 n7 p554-59. October 1981.

Data from the second mathematics assessment of the National Assessment of Educational Progress shows little difference between males and females in overall mathematics achievement at ages 9 and 13. However, at age 17, females are not achieving as well in mathematics as males.

Fox, Lynn. **Mathematically Able Girls: A Special Challenge.** Arithmetic Teacher, v28 n6 p22-23. February 1981.

The special needs and problems associated with encouraging mathematically able girls to pursue advanced training and education are detailed. Several specific suggestions on ways to help female students in these areas are given.

Jones, Peter L. **Learning Mathematics in a Second Language: A Problem with More and Less.** Educational Studies in Mathematics; v13 n3 p269-87. August 1982.

Students in Papua, New Guinea were given a test in which the words more and less were used in different contexts. Half the pupils studied had English as their first language; for the other half English was a second language. Study results of pupil errors suggested serious educational implications.

Kirschner, Vicky. **What Research Says: Females and Mathematics.** School Science and Mathematics, v82 n6 p511-13. October 1982.

Selected mathematics education research on the role and nature of women in mathematics is reviewed. Studies are quoted which indicate that there are no inherent factors which keep girls from learning mathematics at the same level as boys, and that intervention programs can be effective.

Kolata, Gina Bari. **Math and Sex: Are Girls Born with Less Ability?** Science, v210 n4475 p1234-35. December 1980.

The idea that females may be born with less mathematical ability than males is explored. Data from Benbow and Stanley, as well as from the Education Commission of the States, are considered.

Luchins, Edith H. **Women and Mathematics: Fact and Fiction.** American Mathematical Monthly, v88 n6 p413-19. June through July 1981.

Frequently cited sex differences in general psychology texts are noted. With such "differences" specified even when scientific evidence did not exist. Research on brain lateralization and sex difference is reviewed.

Mitchelmore, Michael C. **Three-Dimensional Geomtrical Drawing in in Three Cultures.** Educational Studies in Mathematics, v11 n2 p205-16. May 1980.

Significant differences between groups of students from the U.S., England, and Jamaica in the area of spatial ability were indicated. Some significant differences between sexes were also found on the test used.

Naiman, Adeline. **Women Technophobia and Computers.** Classroom Computer News, v2 n3 p23-24. January-February 1982.

Microcomputers are viewed as bridges between the traditional world defined for women and the male-dominated world of technology. The view expressed is that female use of computers helps to lessen fear of technology and microcomputers can be a beginning supportive step.

Schofield, Hilary L. **Sex, Grade Level, and the Relationship between Mathematics Attitude and Achievement in Children.** Journal of Education Research, v75 n5 p280-84. May-June 1982.

Data on the mathematics attitudes and achievement of fourth through sixth grade students were considered in relation to sex of student, grade level, type of achievement test, and time during the school year at which the measurements were taken.

Taylor, B. Ross. **Equity in Mathematics: A Case Study.** Mathematics Teachers; v76 n1 p12-17. January 1983.

How the Minneapolis Public Schools have addressed equity issues of increasing female and minority participation in mathematics is described. The Minneapolis experience has shown that considerable progress can be made as a result of awareness of inequities and a belief that they can be reduced.

Tomizuka, C.; and Others. **Mathematical Ability: Is Sex A Factor?** Science, v12 n4491 p114-21. April 1981.

Readers responses to an article by Benbow and Stanley in the December, 1980 issue are presented. The letters are followed by a response by the authors of the original article, who clarify points and stress the magnitude of the sex difference uncovered in their research.

OTHER ARTICLES

Anick, C. M., Carpenter, T. M., and Smith, C. **Minorities and Mathematics: Results from the National Assessment of Educational Progress.** Mathematics Teachers, 1982, vol. 74, pp. 560-568

Blum, L. and Givant, S. **Increasing the Participation of Women in Fields that Use Mathematics.** American Mathematical Monthly. December 1980, pp. 785-792.

Cheong, George. **Relations Among Symmetry, Asymmetry, Perceptual Comprehension of Numerals by Kindergarten and First Grade Children.** British Journal of Psychology. V71; pp. 275-82, May 1980.

Elliott, Portia C. **Going "Back to Basics" in Mathematics Won't Prove Who's "Right" but Who's "Left" (Brain Duality and Mathematics Learning.)** International Journal of Mathematical Education in Science and Technology. v11, n2, pp. 213-19, April-June, 1980.

Fauth, G. C. and Jacobs, J. E. **Equity in Mathematics Education: The Educational Leader's Role.** Educational Leadership. March 1980, pp. 485-490.

Fox, Lynn H., Brody, L., and Tobin, D. (eds.) **Women and the Mathematical Mystique.** Baltimore, MD: The Johns Hopkins University Press, 1980.

Gregory, R. J. and others. **Left-Handedness and Spatial Reasoning Abilities: The Deficit Hypothesis Revisted.** Intelligence. v4, n2, pp. 157-59. April-June, 1980.

Townes, B. D., and Others. **Neuropsychological Correlates of Academic Success among Elementary School Children.** Journal of Consulting and Clinical Psychology. v.48, n6, pp. 675-84. December 1980.

ERIC DOCUMENTS--ABSTRACTS

(Call KNOW-NET Project, (206) 753-2858, for assistance in ordering full documents.)

ED191699
Salient Issues in Mathematics Education Research for Minorities. Proceedings from an NIE Sponsored Meeting (Seattle, Washington, April 18, 1980). April 1980. 42pp.

The six brief papers in this document were prepared for an NIE-sponsored meeting in April 1980. Claudette Bradley poses questions (but no answers) on factors affecting American Indians. Alberta Castaneda stresses the need to ascertain how young children learn mathematical ideas. Tony Alfredo Gallegos notes inadequacies of Spanish bilingual programs. Dora Serna advocates the use of students' vernacular language in instruction. Hilda Serna also comments on bilingual concerns. Sau-Lim Tsang presents a review of research studies with Chinese American students, covering the topics of mathematical achievement, testing, Piagetian tasks, and curriculum.

ED194319
Creative Associates, Inc. **State-of-the-Art Report on Mathematics Achievement of Students of Black, Hispanic, and Native American Origins. Final Report and Annotated Bibliography—Selected Research Studies.** July 1980. 88pp.

This two-part report reviews three major areas of influence in the mathematics learning of minority children: (1) learning style, personality and self-concept influences; (2) linguistic effects; and (3) school influences. Low mathematics achievement and nonparticipation of Black, Hispanic, and Native American children have become concerns of administrators, teachers, curriculum directors, and parents. As our society becomes increasingly more technological, the ramifications of this educational situation become more drastic. This paper reviews results of general studies on achievement of minority students and draws on research in mathematics achievement of the general population to locate the impact of certain factors on the minority pupil situation. Each of the three major factors identified are reviewed in detail, with the many interconnected factors discussed, and separate and combination affects assessed where possible. The report concludes with a summary and recommendations for changes in educational practices and suggestions for further research. The appendix contains a list of additional references on other factors relevant to mathematics learning of minority students. Part two of this report is an annotated bibliography of selected research studies in minority education.

ED194346
Junghans, Barbara J. **Math Anxiety, Math Avoidance, Participation in Math: A Summary of Research Curriculum. Recommendations for Career Education.** July 1980. 27pp.

Presented is the product of a curriculum report originated in the Mequon-Theinsville Sex Equity Committee, and focusing on mathematics anxiety and avoidance. The project's two major purposes were to: (1) have the district's two major instructors become aware of the problem, its implications, consequences, and suggested intervention strategies; and (2) initiate the development of some concrete curriculum activities at the high school level, aimed at increasing pupil participation in mathematics. Sections in the report include: (1) a summary of research on math anxiety, including categorized recommendations for overcoming the problem; (2) statistics from Homestead High School's second semester in the 1979-80 school year; (3) reference materials related to mathematics centers and

resource people contracted by the project (5) sample student activities aimed at career education, with problems geared to particular careers and career profiles; (6) a description of a videotape series developed to provide accurate information about women and mathematics; and (6) a selected bibliography.

ED199105
Kincaid, Marylou Butler and Austin-Martin, George. **Relationship Between Math Attitudes and Achievement, Parents' Occupation, and Math Anxiety in Female College Freshmen.** January 1981. 27pp.

A paper presented at the annual meeting of the Southwest Educational Research Association in Dallas, Texas.

ED201478
Lindquist, Mary Montgomery, Ed. **Selected Issues in Mathematics Education.** 1980. 268pp.

This document examines specifically relevant issues and offers useful guidance in making decisions for elementary and secondary school mathematics education in the 1980's. The expertise of 19 authors is reflected in 14 chapters that provide for an overview of mathematics today and recommendations for the future. The individual chapters are titled: (1) Assessing the Mathematics Curriculum Today; (2) Problem Solving: Is It a Problem?; (3) The Role of Computation; (4) Measurement: How Much?; (5) Knowing Rational Numbers; Ideas and Symbols; (6) Probability and Statistics; Today's Ciphering; (7) Geometry: What Shape for a Comprehensive Balanced Curriculum?; (8) The Role of Manipulative Materials in the Learning of Mathematical Concepts; (9) Computers and Calculators in the Mathematics Classroom; (10) Attitudes and Mathematics; (11) Sex-Related Issues in Mathematics Education; (12) The Teacher Variable in Mathematics Instruction; (13) Basic Mathematics Skills; and (14) Recommendations for School Mathematics Programs of the 1980's.

ED201492
Suydam, Marilyn N., Ed. and Kasten, Margaret L., Ed. **Investigations in Mathematics Education, Vol. 14, No. 2.** 1981. 72pp.

Twelve research reports related to mathematics education are abstracted and analyzed. Three of the reports deal with aspects of learning theory, three with topics in mathematics instruction (fractions, problem solving, and application orientation), two with aspects of computer assisted instruction, and one each with advanced placement calculators, mathematics anxiety, and sex differences. Research related to mathematics education which was reported in CIJE and RIE between October and December 1980 is listed.

ED202696
Clements, M. A. **Spatial Ability, Visual Imagery, and Mathematical Learning.** April 1981. 57pp.

This document briefly reviews four areas of educational inquiry. The first section is concerned with definitions of the terms "spatial ability" and "visual imagery"; the second is concerned with training studies in which attempts have been made to improve spatial ability or to encourage greater

use of visual imagery in problem solving; the third pertains to sex differences in spatial performance; and the fourth is concerned with studies investigating relationships between spatial ability, visual imagery and mathematical performance. It is concluded that not only is there little agreement, at present, about how spatial ability and visual imagery should be defined, but also clear relationships between spatial ability and visual imagery, whatever they are, and mathematical learnings have not been identified.

ED204153
Creswell, John L. and Exezidis, Roxane H. **Sex and Ethnic Differences in Mathematics Achievement of Black and Mexican-American Adolescents.** 1981. 7pp.

A paper presented at the annual meeting of the Southwest Educational Research Association in Dallas, Texas.

ED204410
Meece, Judith L. **Sex Differences in Achievement-Related Affect.** April 1981. 21pp.

A paper presented at the annual meeting of the American Educational research Association in Los Angeles. Includes mathematics education and mathematics anxiety.

ED205385
Connor, Jane M. and Serbin, Lisa A. **Mathematics, Visual-Spatial Ability, and Sex Roles. Final Report.** 1980. 75p.

Several studies that were directed towards examining the influence of cognitive factors in sex-related differences in mathematics achievement are reported. Specifically, two lines of research were pursued examining: (1) the relationship between different types of visual-spatial skill and mathematics achievement, and (2) the trainability of visual-spatial skill in junior high school students. Part I of this report discusses two studies that examine the relationship between mathematics achievement (including computation, algebra, and geometry) and visual-spatial skill. The results of the two studies indicated that visual skill and spatial orientation skill are somewhat distinct and both contribute to predicting mathematics achievement. Further research examining the development and trainability of these skills is promoted. Part II focuses on visual-spatial training research that was carried on in 1977-1980. The 1979-1980 investigation is the focus, with the two earlier years of work ending with negative results. This earlier work is documented in the hope that others may profit in reading about these experiences and conclusions.

ED206460
Fennema, Elizabeth, Ed. and Others. **Mathematics Education Research: Implications for the 80's.** 1981. 185pp.

Sponsored by the Association for Supervision and Curriculum Development and the National Council of Teachers of Mathematics this document is available from the Association for Supervision and Curriculum Development, 225 N. Washington St., Alexandria, VA 22314. Cost is $6.75.

ED207847
Hackett, Gail. **Mathematics Self-Efficacy and the Consideration of Math-Related Majors: A Preliminary Path Model.** August 1981. 31pp.

The purpose of this study was to test the hypothesis that mathematics-related self-efficacy mediates the effects of gender and mathematical preparation and achievement on mathematics-relatedness of college major. The responses of 117 undergraduates to a series of inventories and questionnaires yielded seven variables descriptive of the mathematics-related career-choice process; a casual model of the interrelationships of these variables was constructed from predictions based on self-efficacy theory. A path analysis and consequent refinement of the model resulted in a final path model which was congruent with a self-efficacy approach to women's career development. Gender was found to influence mathematics self-efficacy indirectly through two avenues of influence: (1) socialization influences, as captured by the Bem Sex Role Inventory masculinity score; and (2) mathematics preparation, as mediated by the years of high school mathematics and mathematics achievement level. Mathematics-related self-efficacy in turn influenced both mathematics anxiety and mathematics-relatedness of college major. Gender, years of high school mathematics, and mathematics anxiety were also found to influence mathematics major choice directly, as well as indirectly through mathematics self-efficacy. Unexpected results and implications of the model are discussed.

ED209069
Ford Foundation. **Minorities and Mathematics. A Ford Foundation Staff Paper No. 415.** August 1981. 29pp.

This report opens with a bleak picture of the kind of mathematics now taught within American classrooms. The situation for minority students is viewed as particularly grim. The Ford Foundation has launched a major national effort to improve minority students' performance in mathematics and to help mathematics teachers improve the quality of their instruction. The Foundation expects that the lessons learned and the techniques developed will help advance learning in science and mathematics for all students. Nine grants made recently as part of the Foundation's initiative into this area are profiled in the remainder of the document.

ED210153
Ortiz-Franco, Luis. **First Glances at Language and Culture in Mathematics Education.** August 1980. 12pp.

This document highlights points made in papers presented in the two scheduled sessions devoted to issues in mathematics education affecting minority students at the 1980 National Council of Teachers of Mathematics (NCTM) conference. The first meeting was part of the official NCTM conference program. The second meeting was sponsored by the National Institute of Education (NIE). Points from a total of three papers presented over the two meetings are discussed. A paper concerned with verbal problem solving in mathematics among Chicano students is the first to be summarized, as it was only discussed at the NCTM session. The other two were further discussed at the NIE-sponsored meeting. Broad areas of ideas from both these papers are combined in a summary that review the following topic areas: bilingual education, curriculum, teaching and

teacher training, ethnography, testing, and basic research. It is noted that there is a general lack of replications of research studies with minority students on findings and practices that are from mathematics education research that focus on the majority. It is felt the absence of such replications adds to the dubiousness of purportedly generalized outcomes, and needs to be remedied.

ED210154
Ortiz-Franco, Luis. **A Synthesis of Selected Research at NIE in Mathematics Education Related to Minorities.** October 1981. 18pp.

Nineteen projects related to mathematics education issues among minorities, funded by the Learning and Development Unit at the National Institute of Education (NIE), are summarized. It is felt that minorities and mathematics education issues have been largely neglected in recent investigations. This neglect is seen to have the potential to render tenuous the generalizability of empirical findings which have been observed among the non-minority student population. The studies abstracted here cover seven ethnic groups, with both sexes included. Collectively, they include populations in all levels of schooling. More specifically, two investigations include pre-elementary school populations, six involve grades K-6, six focus on grades 7-9, four on grades 10-12, seven include college populations, and five involve adult professionals. Further, three studies do not specify what ethnic groups are included. Of the remaining, eight involve whites, seven pertain to blacks, and four include Puerto Ricans. Each of the groups American Indians, Chinese, and Chicanos is included in two studies, and one report includes Alaskan Natives. One study covers females only, the remaining include both sexes.

ED211353
Fox, Lynn H. **The Problem of Women and Mathematics.** August 1981. 38pp.

This report is divided into two main parts. The first part, The Problem of Women and Mathematics, emphasizes results of ten research projects initially funded by the National Institute of Education (NIE) in 1979. They are emphasized because the findings are current and the projects touched on all major issues. This part has sections on Sex Differences in Mathematics, Factors Influencing the Study and Learning in Mathematics, and Factors Influencing Career Interests and Choice. The second section, Where Do We Go From Here, is drawn from the book "Women and the Mathematical Mystique." This part suggests directions for research and directions for change. The document concludes with a list of references.

ED212466
Stewart, Claire. **Sex Differences in Mathematics. Research Report.** February 1981. 18pp.

This document reviews current literature, and contains the following section titles: General background; Current position in New Zealand and trends; Cross-cultural studies; Spatial visualization and problem solving; Achievement; Attitudes; Attitudes and Achievement; Mathematics as useful; Confidence in mathematics ability; socialization/sex typing; Women in mathematics-related fields; Home environment; School environment; Type of

school; Careers; and Summary. It is noted that the studies vary in their reports of exactly when sex differences in mathematics achievement emerge, but generally it is reported that there are few differences at the primary level. It is in high school that males pull ahead. Even when participation and achievement are almost equal in middle high school, as is the case for recent New Zealand school certificate exams, girls are still less likely than boys to continue with mathematics. It is felt that useful investigations could look further into the relationship between the development of attitudes towards mathematics and participation and achievement in mathematics.

ED219237
Fennema, Elizabeth; Reyes Laurie H. **Teacher/Peer Influences on Sex Differences in Mathematics Confidence. Final Report.** October 1981. 227pp.

The participation in certain classroom processes by students of high and low mathematics confidence who scored above the mean in mathematics achievement was studied over 2 years. The processes selected were: (1) specified types of teacher-pupil interactions, and (2) student engaged time in high or low cognitive level mathematical activities, spatial activities, and with peers. Eighty-two seventh graders were observed daily in their regular mathematics classes for 3 to 4 weeks during the spring semester of 1980, and were again observed in 1981. Between 3 and 14 target students were within each observed class. Roughly equal numbers of each sex were chosen based on the following characteristics for the sample: (1) mathematics achievement scores were at or above the sixth-grade mean of four middle schools, and (2) confidence in mathematics scores were in either the top or bottom quarter for all pupils who had achieved higher than the mean. Two trained observers recorded data on target student and teacher behavior in each classroom. Data were collapsed across classrooms and analyzed using analysis of variance, with sex, confidence level, and years as factors.

ED222037
Fox, Lynn H. **The Study of Social Process That Inhibit or Enchance the Development of Competence and Interest in Mathematics Among Highly Able Young Women, Final Report.** September 1979 through January 1982.

The Study investigated social process and sex differences that might inhibit or enhance the development of interest, self confidence, and competence in the study of mathematics and in the pursuit of careers which require advanced mathematical knowledge and skill among 120 seventh graders, all identified as having superior mathematical ability by the Study of Mathematically Precocious Youth at Johns Hopkins University. A student questionnaire and a parent questionnaire were developed. The Vocational Preference Inventory was administered to students. Questions investigated covered the following areas: characteristics related to family background and aptitude, attitudinal characteristics, support from significant others, home learning, interrelationships between variables, and teacher characteristics. Ss were divided into five groups: A-1 consisted of girls considered to be highly motivated on the basis of their having accelerated their study of mathematics; A-2 included girls who were

considered to be not as highly motivated as A-1; B-1 was a sample of boys considered to be highly motivated; B-2 was a sample of boys considered to be not highly motivated; and C was a sample of girls who appeared to have low interest in mathematics and high interest in the humanities. Among findings were the following: no significant differences were found among the five groups on measures of socio-economic and family constellation variables; A-1 girls had lower levels of self confidence in mathematics than B-1 boys; mothers of boys noticed ability in their sons at a much earlier age than mothers of girls; and most parents of girls felt careers would need to be interrupted for child bearing purposes. Although all the girls were extremely talented in mathematics, they had not been viewed as unusually gifted or unique by the teachers.

ED222326
Schildkamp-Kundiger, Erika, Ed. **An International Review of Gender and Mathematics.**

Because mathematics and gender (sex) is a research area of increasing international interest, a worldwide survey about research concerned with special problems of girls and women when learning mathematics was conducted. Reports included in this document were submitted from Australia, Canada, Dominican Republic, England and Wales, India, Ireland, Israel, New Zealand, and the United States. Points considered in the reports include the general interest in the topic in each country, research concerns (variables other than sex examined, theoretical framework of the research, activities resulting from the research such as programs to increase females' achievement/interest), discussions/developments on the topic, and situations unique to each country. Highlighting information received, it appears that: (1) in most countries the topic is not a central subject of empirical research, although there are some indications that as interest arises, its importance is recognized and leads to research; (2) the United States, followed by England/Wales, is far ahead of other countries in research on mathematics and gender; and (3) that different research strategies are used to understand sex-related achievement differences as explained by sex role perceptions, personality traits, and differentiated interaction processes. References, including articles and documents published in countries participating in the survey, are included. These include background papers, research reports, project information, and teaching suggestions.

OTHER RESOURCES

The following items are available from the Mathematical Association of America, 1529 18th Street NW, Washington, D.C. 20036:

Blacks and Mathematics (BAM) and **Women and Mathematics** (WAM).

Visiting lecturer program to interest Black students or women to pursue mathematics-based careers. Aimed at high school student.

Mathematics Education of Girls and Young Women.

Positions statement. (April 1980)

Mathematics Education of Girls and Young Women.

Information Resource. (September 1980)

Report on Task Force on Problems in Mathematics Education of Girls and Young Women.

Cost $2.50. (February 1980)

Multiplying Options and Subtracting Bias: An Intervention Program developed by Elizabeth Fennema, et. al.

Four videotapes, one each for teachers, students, counselors, and parents--for purchase only. $125 per tape with manual; $375 for all four tapes.

Available from ERIC/SMEAC Information Center, Ohio State University, 1200 Chambers Road, Columbus, Ohio 43212.

Suydam, M. and Kirschner, V. **Selected References on Mathematical Anxiety, Attitude and Sex Difference in Achievement and Participation,** 1980. ($2.50)

Schildkaump-Kundiger, Erika. **An International Review on Gender and Mathematics,** 1982.

Tobias, Sheila. **Overcoming Math Anxiety.** New York, NY: W. W. Norton 1978.

Paths to Programs for Intervention ($16.00).
Resource Catalogue for Practitioners ($13.00).
Resource Manual for Counselors/Math Instructors ($12.00).
Self-Help Kit for Students ($12.00).

Institute for Study of Anxiety in Learning, Washington School of Psychiatry, 1610 New Hampshire Avenue, NW, Washington, D.C. 20009 (202/667-6380).

ASSOCIATIONS

National Council of Teachers Mathematics, 1906 Association Drive, Reston, VA 22091.

National Association for Equal Opportunity in Higher Education, 2243 Wisconsin Ave. NW, Washington, D.C. 20007 (202/333-3855).

National Association for the Advancement of Colored People, New York, NY.

United Negro College Fund, Inc., New York, NY.

Women and Mathematics (WAM), Dr. Eileen L. Poiani, Department of Mathematics, Saint Peter's College, Jersey City, NJ 07306.

Lectureship Program for High School Students. Currently operating in the following areas: Boston, Chicago, Connecticut, South Florida, New York/New Jersey, Oregon, San Francisco Bay Area.

Association for Women in Mathematics, Women's Research Center, Wellesley College, 828 Street, Wellesley, MA 02181.

Organization comprised mostly of women mathematicians.

CAREER INFORMATION

Careers in Applied Mathematics. (1981). Society for Industrial and Applied Mathematics, 1405 Architects Building, 117 South 17th Street, Philadelphia, PA 19103.

Guide to help high school seniors and college students choose a major.

Mathematics at Work in Society: A Career Awareness Package. (November, 1981). Mathematical Association of America, Dolciani Mathematics Center, 1529 18th St. NW, Washington, D.C. 20036.

Four video cassettes and a project book directed at 8-10th graders. The materials show the role math plays in a wide range of career areas.

Askew, Judy. **The Sky's the Limit in Math-Related Careers.** (1981). Educational Development Center, Newton, MA. $3.00 each.

Forty-four page booklet describing math related jobs in Computers, Engineering, Finance, Education, Research Mathematics, and Statistics. Uses quotes from women in these areas. A resource list of women who may be contacted with questions is given.

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